Lecture#2: Energy loss of electrons in matter and Photon interactions

2 main differences:

- Projectile identical to target electron
- Bremsstrahlung process important since m_e small (on +Ze & e^-)
- Moller scattering, Bhabha scattering, e^+e^- annihilation

$$S_{coll} = K.\ \tfrac{1}{2} \left\{ \ln m\beta^2c^2E/[2I^2(1-\beta^2)] - \ln 2\,[2(1-\beta^2)^{1/2} - 1 + \beta^2] + (1-\beta^2) + 1/8\,[1-(1-\beta^2)^{1/2}]^2 \right\}$$

$$S_{rad} = NEZ(Z+1)e^4\{4 \ln (2E/mc^2) - 4/3\}/[137m^2c^4]$$

$S_{tot} = S_{coll} + S_{rad}$

$E_c = 800$ MeV/(Z+1.2) where $S_{coll} \approx S_{rad}$

Radiation length X_0 defined as

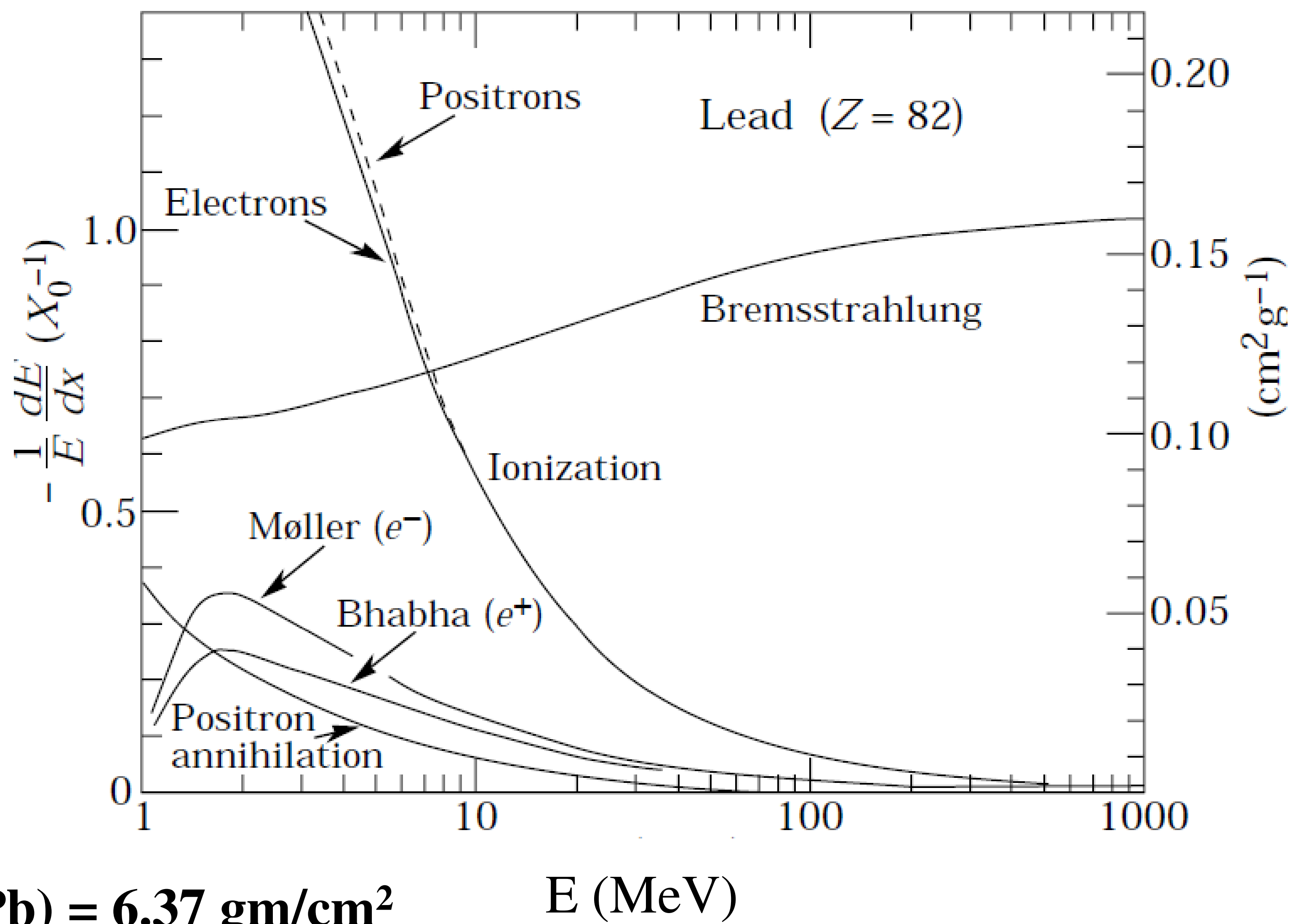
- electron loses energy to 1/e of its initial value by

bremsstrahlung

- 7/9 of γ-attenuation length (or m.f.p.)

$X_0 = 716.4$ A /[Z(Z+1) ln (287/√Z)]   Dahl

Energy loss of e^- and e^+ in Pb



X_0 (Pb) = 6.37 gm/cm^2

Backscattering of low energy electrons

$f_{backsc} = N_e^{backsc}/N_{inc}$

About 0.14, 0.3, 0.5 for Si, Ge, Au respectively at low energy

and at angle of incidence θ_{inc}= 0° (normal to surface)

Drops off at higher energies (about half its value at ~ 6 MeV

Increases with θ_{inc}

Interaction of γ-rays with matter

3 main processes:

1. Photoelectric effect ($\sigma_{pe} \propto Z^5 / E^{3.5}$)
2. Compton scattering ($\sigma_C \propto Z / E$)
3. Pair production $\sigma_{pair} \propto Z^2 \ln(E_\gamma / m_e c^2)$

Photoelectric effect

$A + \gamma \rightarrow A^{+} + e^{-}$

Threshold energy : Binding energy of electron in atom/molecule/solid…

$E_{pe} = E_{\gamma} - BE \Rightarrow$ Full energy deposited in detector (K- or L-Xray escape?)

$\sigma_{pe} = 4\alpha^4 \sqrt{2}\, Z^5 \sigma_0 (mc^2/E_{\gamma})^{7/2}$ where $\sigma_0 = 8\pi r_e^2/3$ for $E_{\gamma} > BE_K$

For E_{γ} higher than, but close to, K-edge

$\sigma_{pe} = (6.3 \times 10^{-18}/Z^2)(BE_K/E_{\gamma})^{8/3}$

Compton scattering

For a photon scattering off a free electron at rest

$\gamma + e^{-} \rightarrow \gamma + e^{-}$

$E_\gamma = E_{\gamma 0}/[1 + E_\gamma/m_e c^2 (1 - \cos\theta)]$

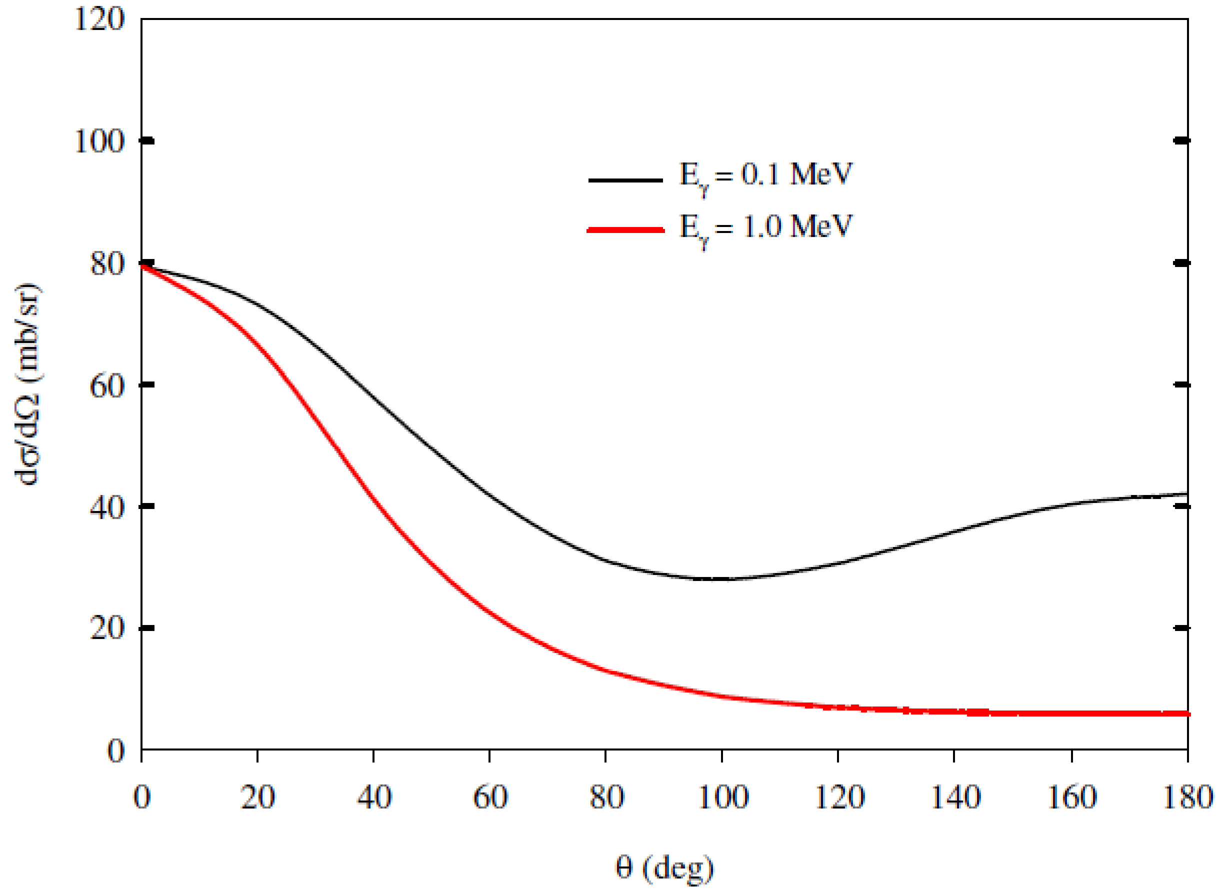
For a bound electron there will be smearing of this energy and will increase the more bound the electron (*used to measure momentum distribution in solids!*)

Limiting cases: 1. If $E_{\gamma 0} << m_e c^2$, $E_\gamma \approx E_{\gamma 0}$ at all angles

2. If If $E_{\gamma 0} >> m_e c^2$, $E_\gamma \approx E_{\gamma 0}$ at near 0°, and ½ $m_e c^2$ for $\theta \approx 180°$

Here $E_e \approx E_{\gamma 0} - ½\, m_e c^2$, $dE_e/d\theta \approx 0 \Rightarrow$ *quasi-monoenergetic* γ!

Angular distribution of Compton scattered gamma-rays



$d\sigma/d\Omega = \frac{1}{2} r_e^2 \, \{1/[1+\varepsilon(1-\cos\theta)]^2\}\{1+\cos^2\theta + \{\varepsilon^2(1-\cos\theta)^2\}/\{1+\varepsilon(1-\cos\theta)\}$

where $\varepsilon = E_\gamma/(m_e c^2)$, $r_e = e^2/m_e c^2 = 2.818$ fm is classical electron radius

- Compton scattering leads to continuous energy deposit upto Compton edge $E_\gamma^{CE} = E_{\gamma 0} - m_e c^2/2$
- In large HPGe detectors Compton followed by photoabsorption leads to full energy peak
- Compton scattering cross section also depends on photon and electron spin polarisations (formula above has averaged over both spins and summed over final ones)

An application of Compton effect: Laser backscattered photons

Suppose you shoot a laser beam (in visible spectrum, for example) at an incoming relativistic electron beam

In rest frame of electron, photons are Doppler shifted by factor

$$\gamma(1 - \beta\cos\theta_{e\gamma}) \approx 2\gamma \text{ for } \theta_{e\gamma} = 180°$$

When it backscatters photon, its Doppler shifted to still higher energies in lab, by another factor of $\sim 2\gamma$

$\Rightarrow$Overall increase in energy by $\sim 4\gamma^2$

For $\gamma \sim 10^3$ ($E_e = 500$ MeV) $E_\gamma \sim 10$ MeV

and $\gamma \sim 10^4$ ($E_e = 5$ GeV) $E_\gamma \sim 1$ GeV!

Pair production

Photons with energy > $2m_ec^2$ can produce e^+ e^- pairs

However **energy-momentum** conservation requires this to occur near an electric charge such as a ***nucleus*** or ***electron***

$\gamma + Ze \rightarrow e^+ + e^- + Ze$

$\gamma + e^- \rightarrow e^+ + e^- + e^-$

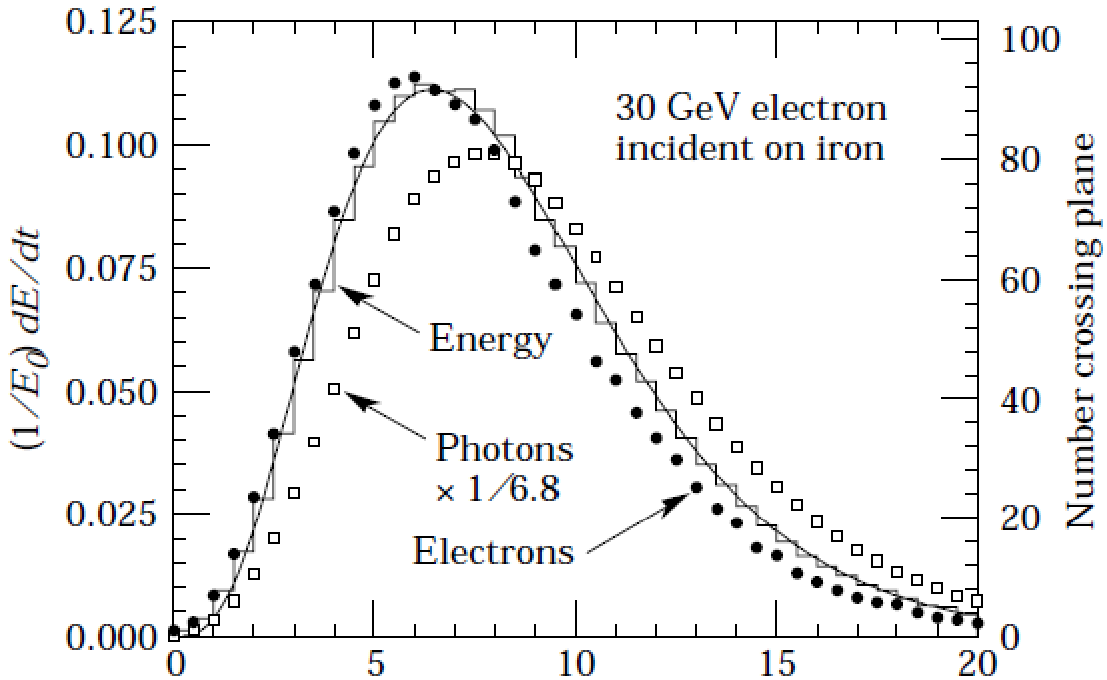
$\sigma_{pair} = 4Z^2 \alpha r_e^{\,2}\ [7/9\{ln\ (2E_\gamma/m_ec^2) - f(Z)\} - 109/54]$

Electromagnetic shower for $E_\gamma >> 1.02$ MeV

Electron-positron pair produced $\rightarrow$ bremsstrahlung $\rightarrow$ $e^+ + e^-$ pair and process repeats (Bhabha – Heitler). First dicussed in the context of cosmic ray showers.

In detector max. energy deposition via photoelectric and pair production processes. Compton process leads to only partial energy deposit. In EM shower 511 keV annihilation photons and bremsstrahling can escape leading to tailing on the lower enrgy side of full energy peak

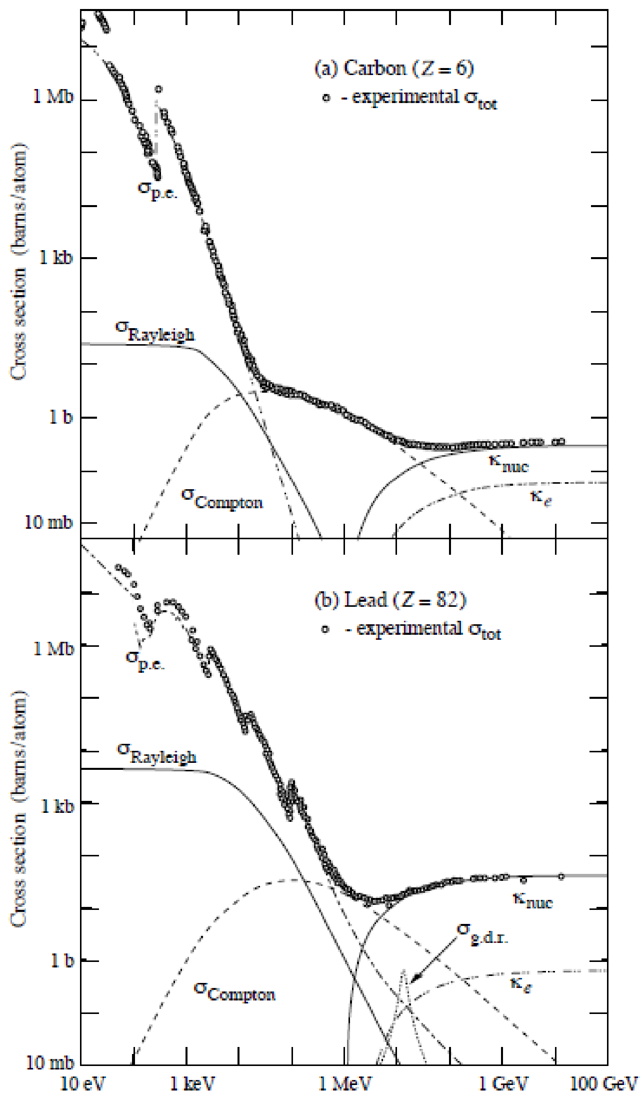
EGS4 simulation of 30 GeV electron in Iron



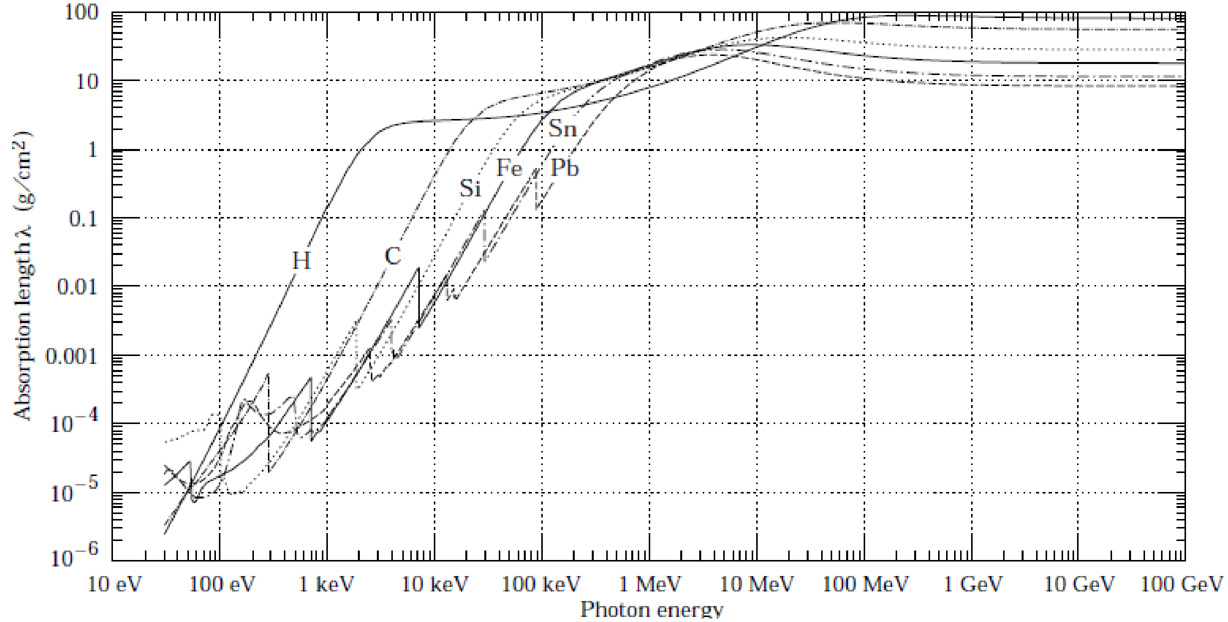
Thickness (in units of X_0)

From RPP 2006

Photon interaction with lead



Photon energy



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