QCD @ hadron colliders

Prakash Mathews

Saha Institute of Nuclear Physics

- **Brief History of Strong Interaction**
- **Essential features of QCD**
  - **Factorisation**
  - **Universality of IR behaviour**
  - **Cancellation of IR singularities**
  - **IR safe observables**
- **Precision QCD at Hadrons collider**
- **Diphoton production at the LHC in TeV scale gravity models**

High Energy Accelerators

- Accelerators are tools to study the structure of matter and interactions at very short distances
- Uses stable particles (and their antiparticle) eg: Protons and electrons
- Mostly circular machines to use the beams for longer duration
- Proton: limited by bending power of dipoles magnets in the ring
- Electron: limited by synchrotron radiation= energy loss per revolution $\delta E \sim (\frac{E}{m})^4(\frac{1}{R})$
- Present machines

Machine	Year	Beams	Energy($\sqrt{s}$)
Tevatron	1987	$p\bar{p}$	1.96 TeV
RHIC	2000	$pp(AA)$	200-500 GeV
LHC	2009	$pp(AA)$	10 $\longrightarrow$ 14 TeV

Advantage of a PP machine to search for new physics is
○ A prior we neither know the mass nor do we know if they exist
○ Hadronic CM energy $S = 14$ TeV, partonic cm energy $s = x_1 x_2 S$ is the energy available for discovery $M^2_{new} \sim s = x_1 x_2 S$, implies $M_{new} \sim 0.2\sqrt{S} \sim 3$ TeV
○ Hadron machine is a natural scanner and good to exploring new physics while a e^+e^- machine is good to study the properties of known physics

CERN Site

Strong Interaction

- Binds quarks and gluons inside hadron and is the strongest of the four fundamental forces in nature

- About 100 times em force, a factor 10^{14} stronger then Weak interaction and a stunning factor of 10^{40} stronger than the Gravitational force

- But experience in the macroscopic world is dominated by gravitational and the em force as the strong and weak are short ranged

- Restriction of the strong force to subatomic distances is a consequence of two features: confinement and asymptotic freedom

- Confinement is a necessary requirement to explain the fact that no isolated quarks have ever been observed in any experiment, although symmetry arguments and scattering experiments in the 1960's established quarks with -1/3 and +2/3 electric charge units and newly introduced quantum property called colour charge

Historical Developments

● Gell-Mann and Zweig realised in 1964 that the whole spectroscopy of hadron could be explained by a small number of quarks if baryons are made out of 3 quarks and meson out of one quark and antiquark

● quarks must hence have 1/3 or 2/3 elementary electric charge units and spin 1/2

● By end of 1960s static picture of quarks as the constituents of hadrons was confirmed through dynamics observed at high $e\,P$ scattering experiments at SLAC

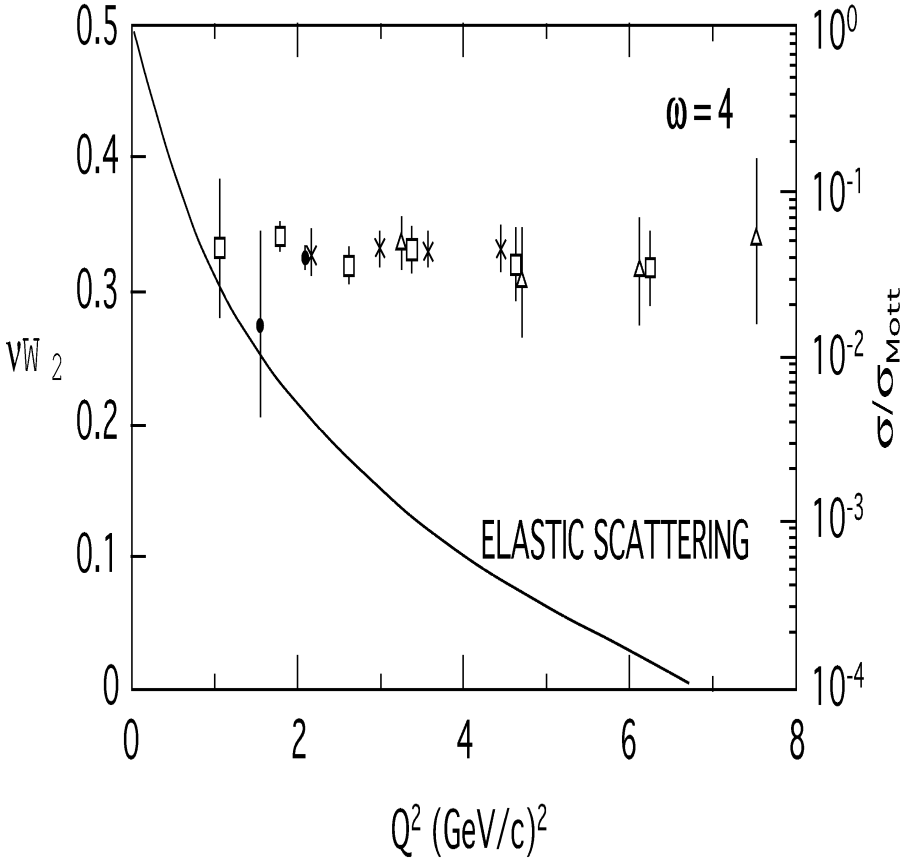
● Instead of decreasing with increasing momentum transfer as expected for elastic scattering of electrons at protons as a whole, the cross section showed a scaling behaviour as it should occur if the electrons scatter on quasi-free, point like and nearly massless constituents inside the proton

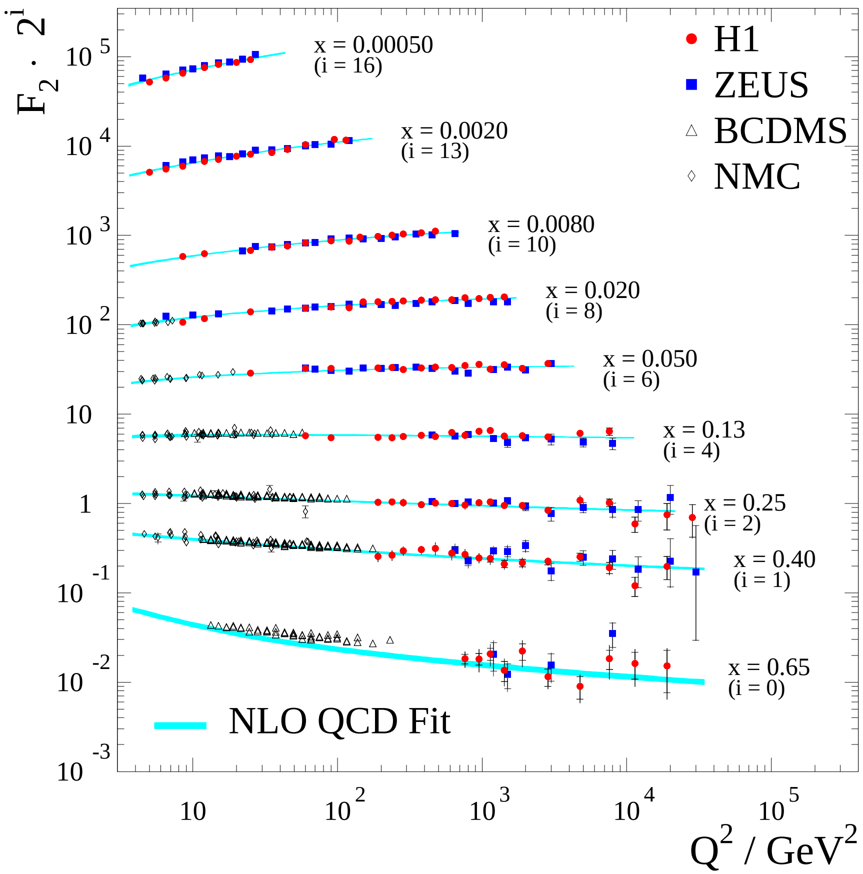
● quark model was very successful in describing the properties, multitude and dynamic behaviour of hadrons, it had severe short comings

○ Violation of the Pauli-principle
○ Prediction of neutral pion lifetime was off by a factor nine
○ No particle of elementary electric charge 1/3 or 2/3 observed in colliders

DIS then and now

• **SLAC (1969): sub structure of nucleon— Nobel 1990: Limited range of x and Q^2 in fixed-target lepton-nucleon scattering experiments, prevented unambiguous test of QCD scaling violations and running of α_s**





• **HERA (2005): extended the range of Q^2 by more than 2 orders of magnitude and the range in x by more then 3 orders of magnitude— precise test of scaling violations and running coupling were achieved**

Chromodynamics

- 3 different colour quantum states for each quark species was introduced and solved the spin-statistics, saved Pauli-principle and explained the missing factor of nine $(= 3^2)$ of the pion lifetime

  M Y Han and Y Nambu (1965)

- Notion that hadrons consist either of 3 quarks (baryons) or a quark and an anti quark (meson) arranged such that the net colour charge of hadron would vanish— could account for the fact that the strong force is short-ranged

- Finally, in early 1970's a field theory of the strong force, Quantum Chromodynamics (QCD), was introduced. Coloured spin-1 particles called gluons, which couple to colour charges of quarks and also to coloured gluons themselves

  H Fritzsch and M Gell-Mann (1972)

  H Fritzsch, M Gell-Mann and H Leutwyler (1973)

- Chromo-Statics turned into Chromo-Dynamics

Asymptotically Freedom

• Symanzik demonstrated (1970) in model theories that charges may change their effective size when they are probed in scattering experiments, at large and at small distances— quantified through Symanzik's β-function

• SLAC data on approximate scaling and the notion of free quarks and gluons inside the proton required a -ive β-function. All field theories probed during that time had a +ive β-function

• Crucial question in the early 1970s was therefore whether QFT was compatible with ultraviolet stability (asymptotic freedom)?

• Majority view was expressed by Zee (1973); conjecturing that "there are no asymptotically free quantum field theories in 4-dim"

• Coleman and Gross set out to prove that conjecture, their graduate students Politzer and Wilczek (with Gross) tried to close a loophole: β-function for nonabelian gauge theories— still unpublished and probably unknown to everybody except t'Hooft

• Politzer and Gross & Wilczek finally demonstrated in 1973 that Chromo-Dynamics, with coloured quarks and gluons, obeying $SU_c(3)$ symmetry, generated a -ive β-function— the quarks and gluons are asymptotically free

Nobel prize 2004

Asymptotically Freedom

- Explained the approximate scaling in the SLAC data at high energies, and at the same time an increase of coupling strength at low energies lead to confinement

- An important consequence of asymptotic freedom is that the strong coupling α_s is small enough, at sufficiently high energies to allow application of perturbation theory in order to provide quantitative predictions of physical processes

- Quantum Chromodynamics now started its triumphal procession as being the field theory of the strong interaction. Many refined calculations theoretical predictions and experimental verifications were ventured

- Asymptotic freedom and or equivalently the existence of colour charged gluons had to be tested, quantified and proven experimentally. The strong coupling parameter, $\alpha_s(Q^2)$ had to be determined and its energy dependence verified to be compatible with asymptotic freedom

Quantum Chromodynamics (QCD)

• QCD is the gauge field theory of the strong interaction and describes the interaction of quarks through the exchange of massless vector gauge bosons

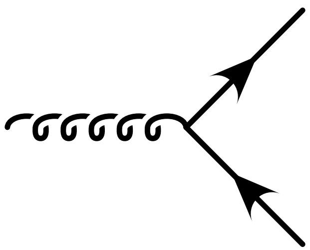
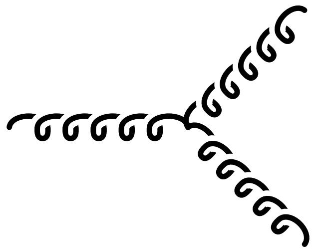
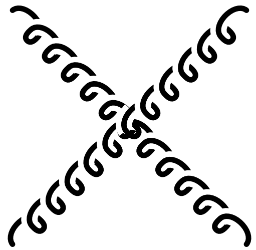
$$
\begin{aligned}
\mathcal{L}_{QCD} &= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \sum_{f=1}^{n_f} \overline{q}_f (i \not{D} - m_f) q_f \\
G^{\mu\nu}_a &= \partial^\mu G^\nu_a - \partial^\nu G^\mu_a - g_s f_{abc} G^\mu_b G^\nu_c \\
(D^\mu)_{ij} &= \delta_{ij} \partial^\mu - i g_s \sum_{a=1}^{8} G^\mu_a T^a_{ij}
\end{aligned}
$$

• QCD Lagrangian is invariant under the local $SU_c(3)$ transformation

$$
\begin{aligned}
q_i \longrightarrow q'_i &= U_{ij}(\varepsilon_a) q_j & U_{ij} &= \exp\{-i T^a_{ij} \varepsilon^a\} \\
G_\mu \longrightarrow G'_\mu &= U(\varepsilon) G_\mu U^\dagger(\varepsilon) + \frac{i}{g_s} \left(\partial_\mu U(\varepsilon)\right) U^\dagger(\varepsilon) & G^\mu_{ij} &= G^\mu_a (T_a)_{ij}
\end{aligned}
$$

• QCD does not predict the actual value of $\alpha_s = g_s^2/4\pi$, however it definitely predicts the functional form of its energy dependence

QCD Feynman rules

Coupling:	g_s	g_s	g_s^2
			
Colour:	$(T_a^F)_{ij}$	$(T_a^A)_{bc} = -if_{abc}$	$f_{abc}f_{abc}$
Lorentz:	γ^μ	$V^{\mu\nu\rho}$	$W^{\mu\nu\rho\sigma}$

- **A theory formulated in terms of quarks and gluons at the Lagrangian level but observed in nature as hadrons**
- **Hadrons can carry definite flavour quantum number and hence the hadronic wave functions are non-singlets under the falvour symmetry $SU(n_f)$ while hadrons do not carry any colour quantum number and hence transform as singlet under $SU_c(3)$ transformation**
  - **Baryons** $\frac{1}{\sqrt{6}} \sum_{ijk} \epsilon_{ijk} q_i^{f_1} q_j^{f_2} q_k^{f_3}$
  - **Mesons** $\frac{1}{\sqrt{3}} \sum_{ij} \delta_{ij} q_i^{f_1} \bar{q}_j^{f_2}$

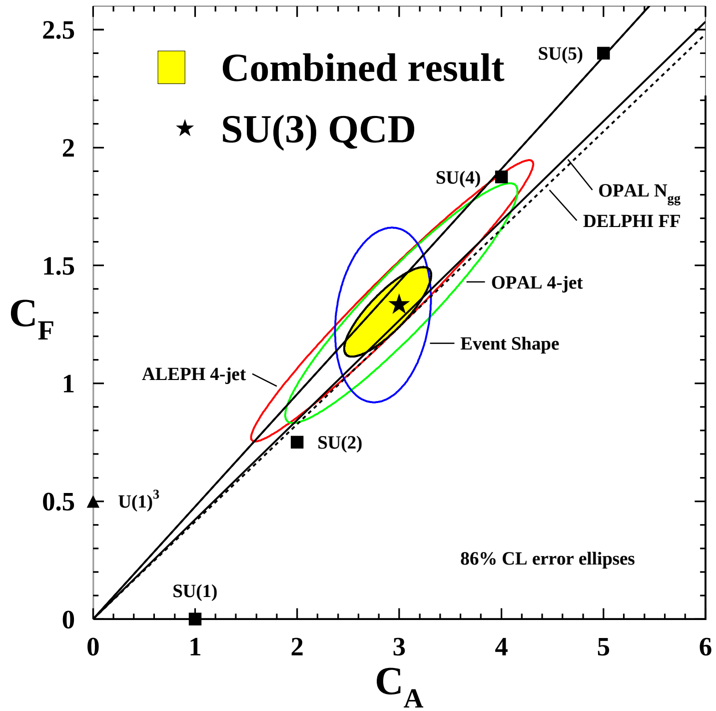
Experimental Group Theory

- Can experimentalists measure all the information contained in the vertices
- Vertices are determined by quark and gluon representation matrices T_a^F and T_a^A (general symmetry group). Combination that appear in measurable quantities are the following traces and sums:

$$tr(T_a^R T_a^R) = T_R \delta_{ab} \qquad \sum_a (T_a^R)_{ij} (T_a^R)_{jk} = C_R \delta_{ij} \qquad (R = F, A)$$

- At LEP, data statistics and precision allowed to actually determine experimentally values of C_A (number of colour charge) and C_F

	SU(3)	LEP
C_F	$\frac{4}{3}$	1.30 ± 0.01 (stat) ± 0.09 (sys)
C_A	3	2.89 ± 0.03 (stat) ± 0.21 (sys)



◦ Excludes theories exhibiting symmetries other than $SU(3)$

Energy dependence of strong coupling

- QCD β function calculated up to 4-loops in the $\overline{\mathrm{MS}}$ scheme

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta(a_s) = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 + \mathcal{O}(a_s^6)$$

$$a_s = \alpha_s/4\pi = g_s^2/16\pi^2, \; g_s = g_s(\mu^2)$$

- 4-loops β function for $N_c = 3$

$$\beta_0 = 11 - \tfrac{2}{3} N_f \qquad \beta_1 = 102 - \tfrac{38}{3} N_f \qquad \beta_2 = \tfrac{2857}{2} - \tfrac{5033}{18} N_f + \tfrac{325}{54} N_f^2$$

$$\beta_3 = \left(\tfrac{149753}{6} + 3564\zeta_3\right) - \left(\tfrac{1078361}{162} + \tfrac{6503}{27}\zeta_3\right) N_f + \left(\tfrac{50065}{162} + \tfrac{6472}{81}\zeta_3\right) N_f^2 + \tfrac{1093}{729} N_f^3$$

Ritbergen, Vermaseren, Larin Phys. Lett. B400 (1997) 379

- Strong coupling g_s of QCD is characterized by two important features:
  - asymptotic freedom $\quad g_s \longrightarrow 0 \quad$ UV
  - confinement $\quad g_s \longrightarrow \infty \quad$ IR

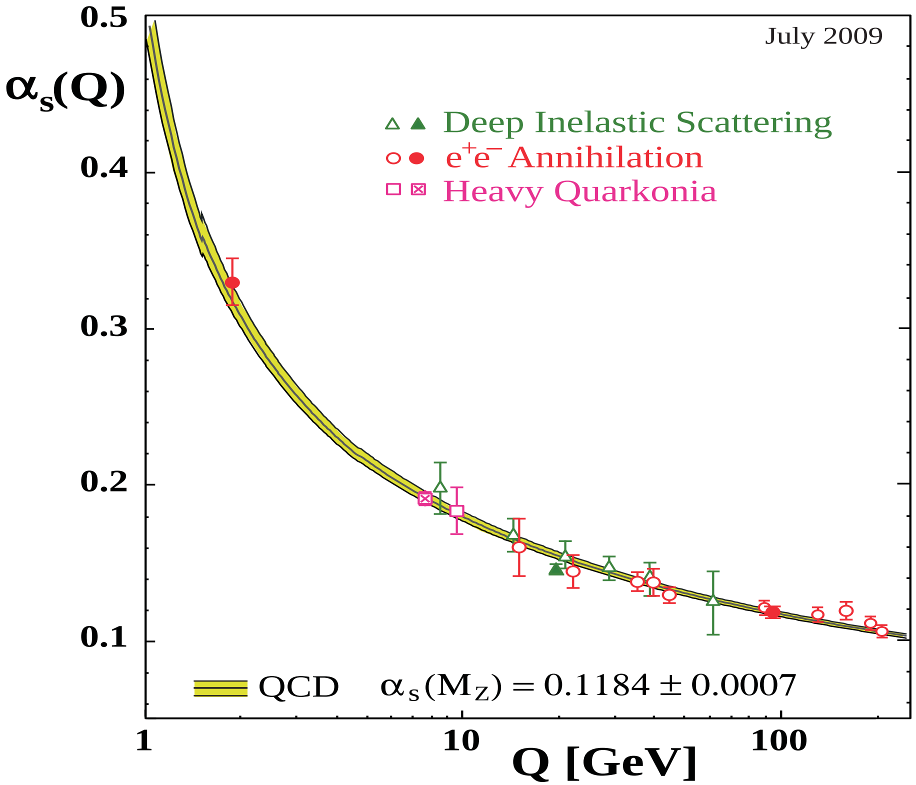
- These properties are strongly dependent on N_f or N_c

Asymptotic freedom in QCD

- **Solution of β-function equation to 1-loop**

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \ln\left(\frac{Q^2}{\mu^2}\right)} \equiv \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)}$$

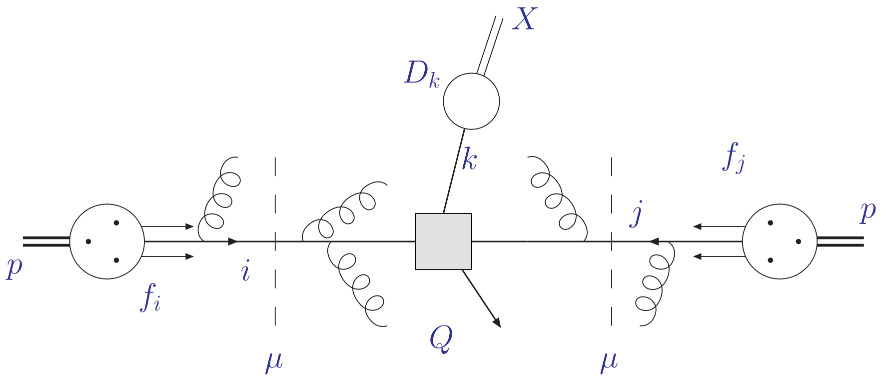
For $N_f < 17$, $\alpha_s(Q^2)$ will asymptotically decrease to zero for $Q^2 \to \infty$



- **Summary of measurements of α_s as a function of the respective energy scale Q**
- **Curves are QCD predictions for the combined world average of $\alpha_s(M_Z)$, in 4-loop approximation and using 3-loop threshold matching at the heavy quark pole masses**
- $\Lambda^{(5)}_{\overline{MS}} = (213 \pm 9) MeV$
- **At high energies asymptotic freedom ensures application of perturbation theory (but $\alpha_s >> \alpha_{QED}$)**

Perturbative QCD at colliders

- Basic prerequisite for the application of perturbative QCD at colliders is factorisation for hard-scattering process



- Hadronic cross section for production of a final state X factorises into products of PDFs $f_{\frac{i}{P}}$ and partonic cross sections $\hat{\sigma}^{ij\to X}$

$$\sigma^{PP->X}(s;\alpha_s,\mu_F,\mu_R) =$$

$$\sum_{ij}\int_0^1 dx_1 dx_2 f_{\frac{i}{P}}(x_1,\alpha_s,\mu_F) f_{\frac{j}{P}}(x_2,\alpha_s,\mu_F)\hat{\sigma}^{ij\to X}(sx_1x_2;\alpha_s,\mu_F,\mu_R)$$

$i, j = q, \bar{q}, q$ are partons carrying a fraction $x_{1,2}$ of the proton momentum

- Truncating the cross section at a given order in perturbation theory induces dependence on factorisation scale μ_F and renormalisation scale μ_R

Parton luminosity at hadron colliders

- PDFs $f_i(x, \mu^2)$ are indispensable ingredients of hard scattering process involving initial state hadrons— not calculable in perturbative QCD

- Universality allows for the determination of PDFs in global fits to experimental data— a non perturbative input

- Independence of any physical observable on scale μ gives rise to evolution equation for PDFs, which is a system of coupled integro-differential equations corresponding to different possible parton splittings

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_{q_i}(x,\mu^2) \\ f_g(x,\mu^2) \end{pmatrix} = \sum_j \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{q_i q_j}(z) & P_{q_i g}(z) \\ P_{g q_j}(z) & P_{gg}(z) \end{pmatrix} \begin{pmatrix} f_{q_j}(x/z,\mu^2) \\ f_g(x/z,\mu^2) \end{pmatrix}$$

- Splitting functions P_{ij} are universal quantities and are calculated in perturbative QCD and has an expansion is α_s

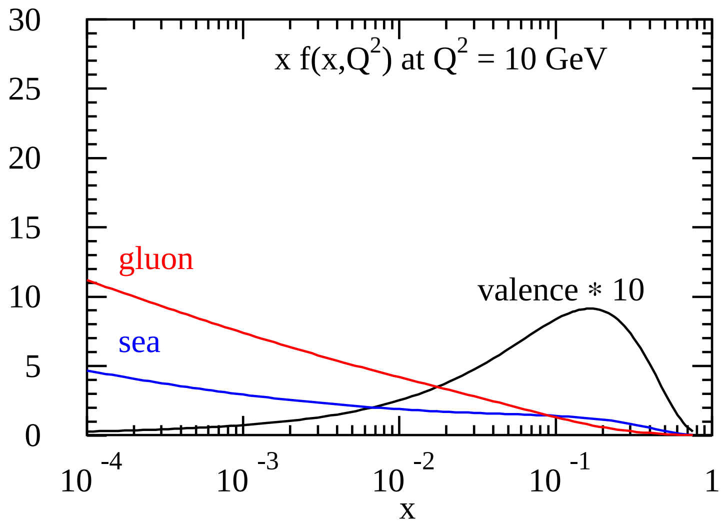
$$P = \alpha_s P^{(0)} + \alpha_s^2 P^{(1)} + \alpha_s^3 P^{(2)} + \cdots$$

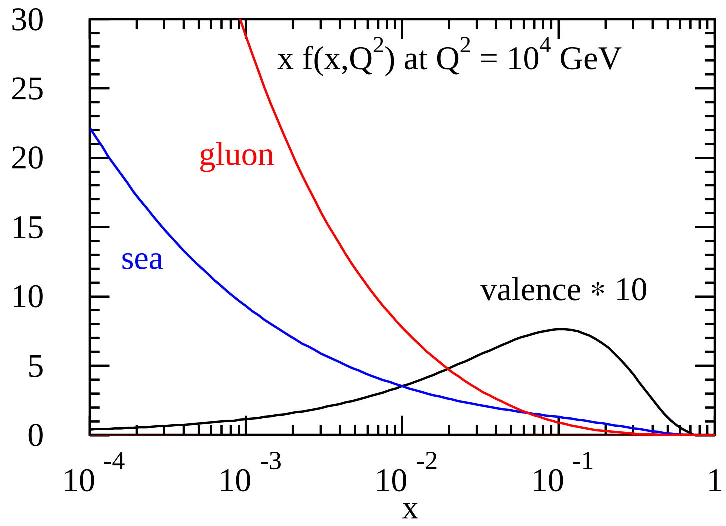
- Currently splitting functions are known to NNLO

Moch, Vermaseren, Vogt 2004; Vogt, Moch, Vermaseren 2004

Parton evolution

• **Given an input distribution at low scale ($Q^2 = 10$ GeV) determined in a global fit to data, the evolution equations can be used to predict the PDFs at LHC energies ($Q^2 = 10^4$ GeV)**





• **PDFs are extracted from experiments in some factorisation scheme, to a particular order, by various groups performing global fits to available data on DIS, DY and other hadronic process**

Partonic cross section

- Partonic cross section perturbatively calculable order by order in α_s

$$\begin{aligned}\hat{\sigma}^{i\,j\to X}(\alpha_s,\mu_F,\mu_R) &= \alpha_s(\mu_R)^{n_\alpha}\Big\{\hat{\sigma}^{(0)}+\frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R)\\ &\quad+\left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)}(\mu_F,\mu_R)+\mathcal{O}(\alpha_s^3)\Big\}\end{aligned}$$

typically if $\mu_R = 100$ GeV, $\alpha_s(\mu_R)\sim 0.1$, then LO term in the expansion $\hat{\sigma}^{(0)}$ would suffice to get a 10 % uncertainty

- However in hadron collider corrections from NLO $\hat{\sigma}^{(1)}$ can increase the cross section by 30 % to 80 %.

LO	qualitative
NLO	quantitative
NNLO	few percent precision

- Any residual scale dependence is a measure of the quality of a given calculation in finite perturbative order

QCD improved Parton Model

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, Q^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, Q^2, \mu_F\right)$$

- **Partonic cross section perturbatively calculable:**

$$d\hat{\sigma}^{ab}\left(z, Q^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R^2)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, Q^2, \mu_F, \mu_R\right)$$

- **Non-perturbative partonic flux:**

$$\Phi_{ab}(x, \mu_F) = \int_x^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{x}{z}, \mu_F\right)$$

- $f_a^{P_1}(x, \mu_F)$ **are Parton distribution functions,** x **is the partonic momentum fraction**
  - μ_R **is the Renormalisation scale**
  - μ_F **is the Factorisation scale**

Source of Uncertainties: Theoretical & Experimental

Theoretical Uncertainties:

- **Renormalisation scale: Due to UV divergence at beyond Leading Order**

$$\alpha_s \longrightarrow \alpha_s(\mu_R^2)$$

- **Factorisation scale: Originate from light quarks and massless gluon. Parton distribution functions are renormalised at the factorisation scale μ_F**

$$f_a(x) \longrightarrow f_a(x, \mu_F^2) \qquad\qquad a = q, \bar{q}, g$$

- **Observables should be "free" of μ_R and μ_F, but "Fixed order" perturbative results depend on μ_R and μ_F**
- **Can in principle give large uncertainties**

Experimental Uncertainties (PDFs):

Not calculable but extracted from experiments in some factorisation scheme by various groups by global fits to available data on DIS, DY and other hadronic process

IMPORTANT FOR NEW PHYSICS SEARCHES TO HAVE BETTER CONTROL OVER THE THEORETICAL UNCERTAINTIES

Mass Factorisation

Divergent NLO correction involve loop and phase space integrals— regularised using dimensional regularisation $n = 4 + \epsilon$

- IR divergences:
  IR divergences appears when $|k| \to 0$, cancels between virtual and real diagrams (Bloch-Nordsieck Theorem)
- Collinear divergences:
  • Final state collinear: singularities cancel if we consider a final state inclusive process summing over all experimentally indistinguishable final states (KLM Theorem)
  • Initial state collinear: singularities left over— leads to mass factorisation

Mass Factorisation Theorem:

- Collinear divergences can be factored out of the sub process cross section
- Mass singularities encountered in QCD are process independent

Universality of the collinear singularities enables a process independent way to absorb them into parton distributions

Politzer NPB 129 (1977) 310 $\cdots$

$\cdots$ Collins, Sopper, Sterman

@ the LHC

- The challenge is to solve master equation

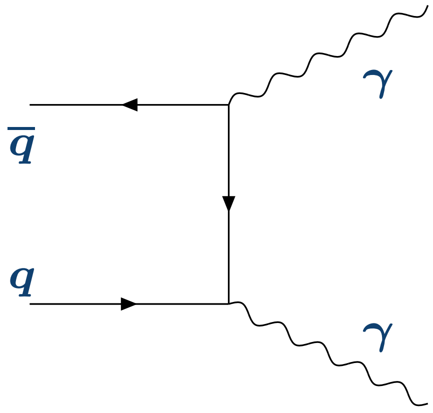
$$\text{new physics} = \text{data} - \text{Standard Model}$$

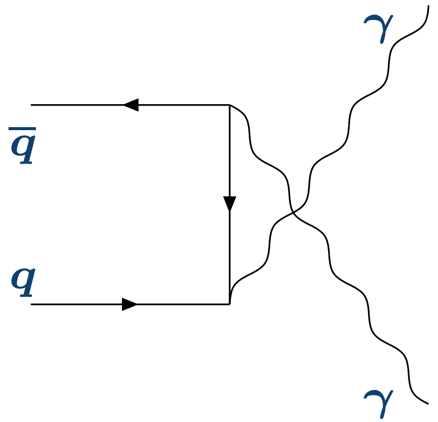
- New physics searches require the understanding of SM background
- LHC explores the energy frontier, theory has to match or exceed accuracy of LHC data
- LHC is a QCD machine, perturbative QCD is an essential and established part of the toolkit
- Asymptotic freedom, factorisation and evolution are the instruments we use to analyse QCD processes at colliders

Di-photon Process

$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

Leading Order



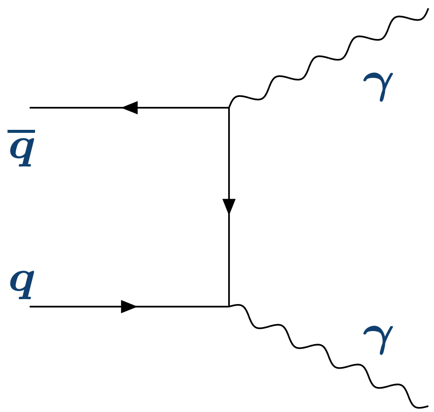


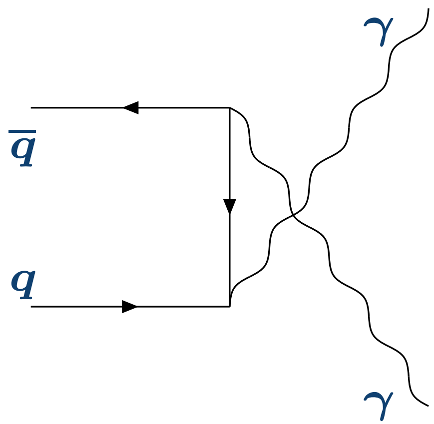
SM

Di-photon Process

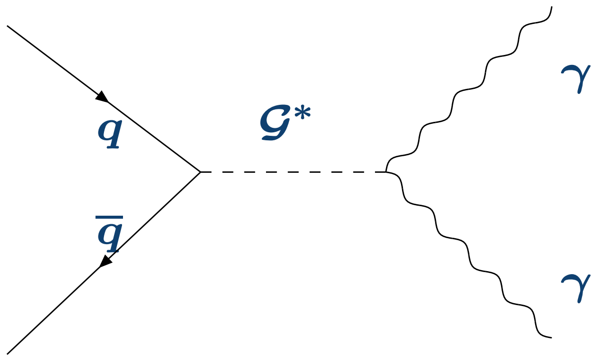
$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

Leading Order





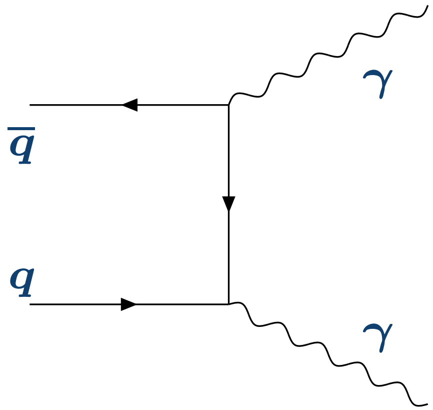
SM

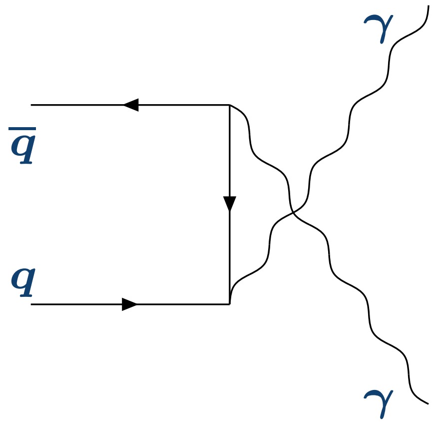


Di-photon Process

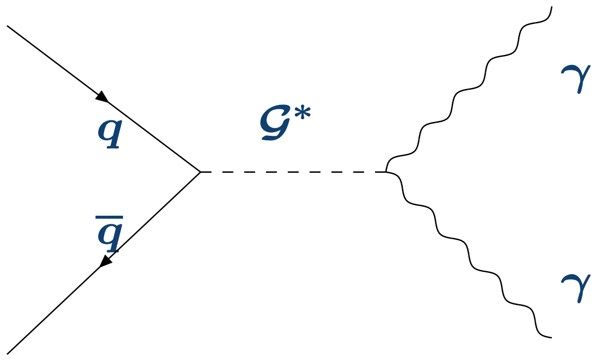
$$P_1(p_1) + P_2(p_2) \longrightarrow \gamma(k_1) + \gamma(k_2) + X$$

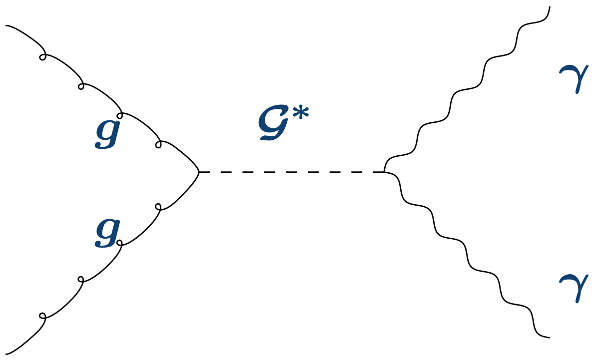
Leading Order





SM





$Gravity$

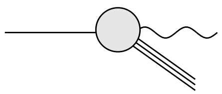
Di-photon Process

- Prompt photons with large transverse momenta at hadron colliders is an interesting laboratory of the short distance dynamics of quarks and gluons and is an important channel for Higgs searches in the mass range 80 GeV $\leq m_H \leq$ 140 GeV and various BSM studies

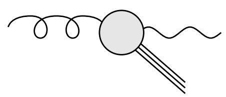
- "Prompt photons" means they do not come from decay of hadron (π^0, η etc). Photon are faked by hadron, for eg: π^0s at large p_T could go into two nearly collinear photons which are difficult to distinguish from a single photon

- Prompt photon could be classified as (a) direct, both photons are not as a result of fragmentation and (b) fragmentation, atleast one of the photon is as a result of fragmentation

$D_{\frac{\gamma}{q}}(z)$



$D_{\frac{\gamma}{g}}(z)$



- At colliders secondary photons coming from the decay of hadron overwhelms the prompt photons signal but secondary photons can be rejected by experimental selection of prompt photons using isolation cuts

Contributing Subprocess to digamma production

Leading Order:

Standard Model	KK-Modes
$q + \bar{q} \rightarrow \gamma\gamma$	$q + \bar{q} \rightarrow \mathcal{G}$ $g + g \rightarrow \mathcal{G}$

Next-to-Leading Order:

Standard Model	KK-Modes
$q + \bar{q} \rightarrow \gamma\gamma + g$, $q + \bar{q} \rightarrow \gamma\gamma$ + one loop $q + g \rightarrow \gamma\gamma + q$, $\bar{q} + g \rightarrow \gamma\gamma + \bar{q}$	$q + \bar{q} \rightarrow \mathcal{G} + g$, $q + \bar{q} \rightarrow \mathcal{G}$ + one loop $q + g \rightarrow \mathcal{G} + q$, $\bar{q} + g \rightarrow \mathcal{G} + \bar{q}$ $g + g \rightarrow \mathcal{G} + g$, $g + g \rightarrow \mathcal{G}$ + one loop

Phys. Lett. B 672 (2009) 45; Nucl. Phys. B818 (2009) 28, *with* MC Kumar, V Ravindran & A Tripathi

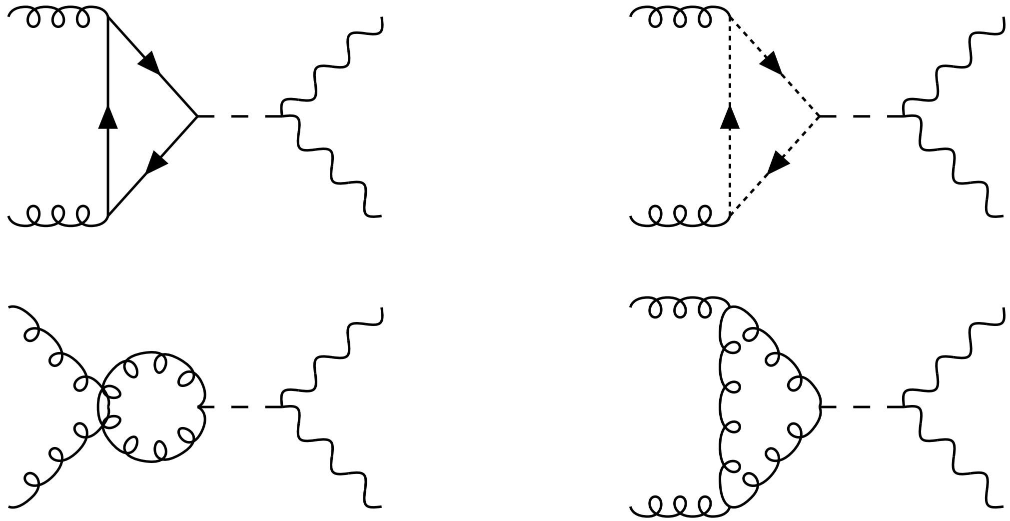
Virtual Contributions $q\,\bar{q} \to \gamma\,\gamma$

$\mathcal{O}(\alpha_s)$ virtual corrections comes from the interference between the virtual graphs of the (SM + BSM) and the (SM + BSM) Born graphs

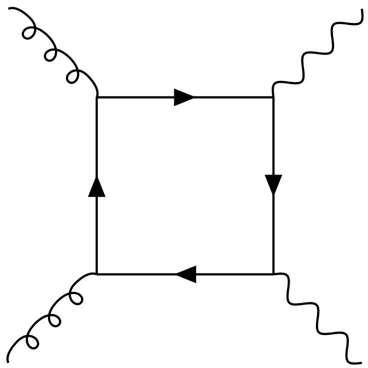
- $\boldsymbol{\mathcal{O}\Big(g_s^2(e_q^4 + e_q^2\kappa^2 + \kappa^4)\Big)}$

Virtual contributions $g\,g \to \gamma\,\gamma$

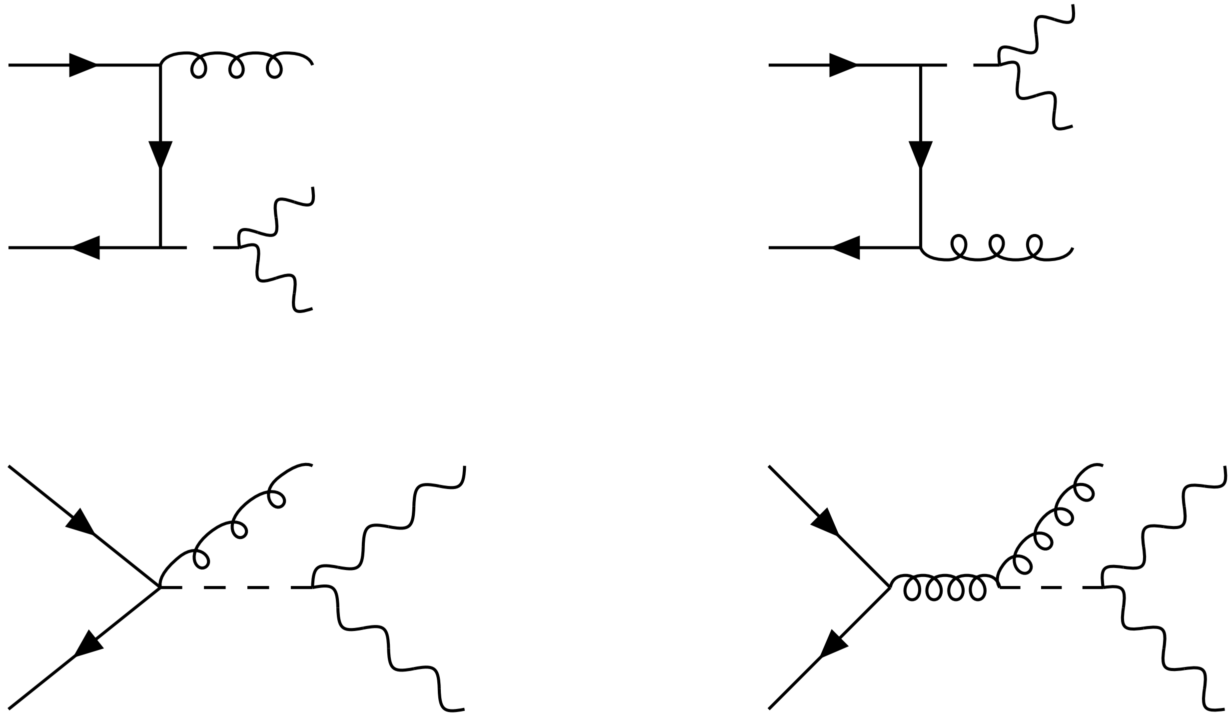
- $\mathcal{O}\left(g_s^2\ \kappa^4\right)$



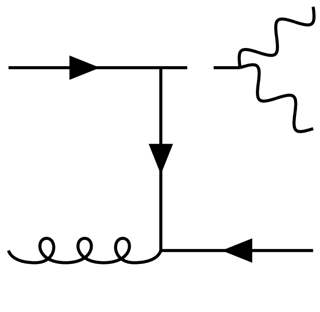
- SM gluon fusion via quark loop $\mathcal{O}\left(g_s^2 e_q^2 \kappa^2\right)$

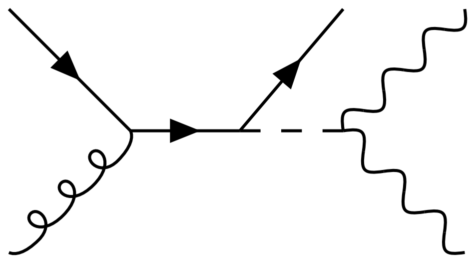


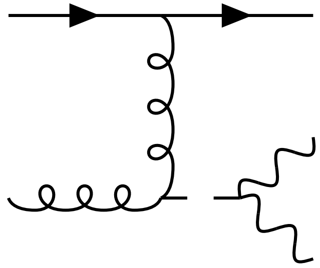
Real Contributions $q\ \bar{q} \longrightarrow \gamma\ \gamma\ g$

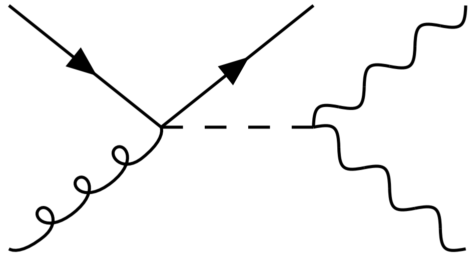


Real Contributions $q\ g \rightarrow q\ \gamma\ \gamma$

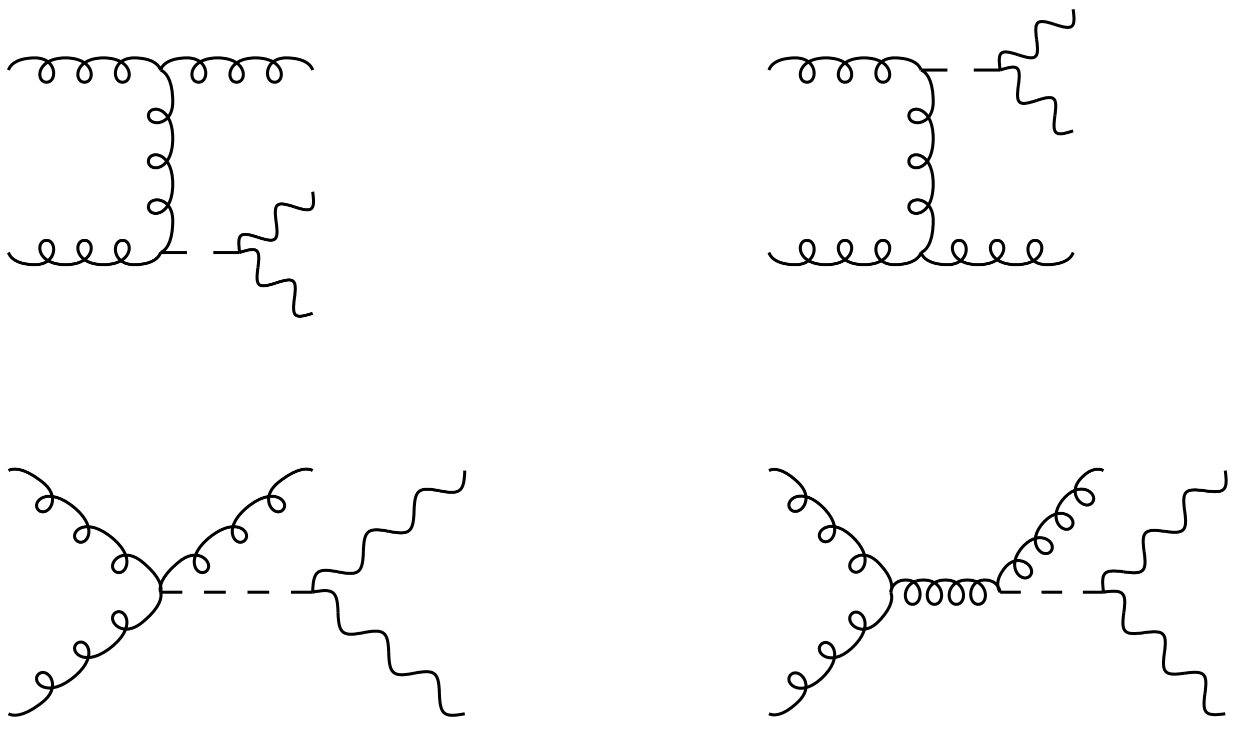








Real Contributions $g\, g \to g\, \gamma\, \gamma$



Phase-space slicing method with *two cutoffs*

- Isolating 3-body phase space regions where soft and collinear singularities occur— impose arbitrary boundaries by introducing $small$ cut-off parameters δ_s, δ_c

		soft		hard		
		$\overbrace{0 \le E_g \le \delta_s \frac{\sqrt{s_{ab}}}{2}}$		$E_g > \delta_s \frac{\sqrt{s_{ab}}}{2}$		
$d\sigma_{ab}^{real}$	=	$d\sigma_{ab}^{real}(\delta_s)$	+	$d\sigma_{ab}^{real}(\delta_s)$		
$d\sigma_{ab}^{real}$	=	$d\sigma_{ab}^{real}(\delta_s)$	+	$d\sigma_{ab}^{real}(\delta_s, \delta_c)$	+	$d\sigma_{ab,fin}^{real}(\delta_s, \delta_c)$
				$0 \le t_{ij} \le \delta_c s_{ab}$		$t_{ij} > \delta_c s_{ab}$
				hard collinear		hard non collinear

- Phase space integration on the mutually exclusive soft and collinear region are performed not on the full matrix element but in the leading pole approximation of soft and collinear region

- Now only the logarithms of the cut-off parameters are retained and all positive powers of the cut-off parameters are set to zero

Dependence on the cutoff parameters δ_s and δ_c

- Performing the phase space integrals in $4+\epsilon$ dimensions, the soft and collinear poles are exposed. Adding the virtual contributions, all double and single poles of soft (IR) origin automatically cancel

- Remaining collinear poles are then factorised in the parton distribution or fragmentation function as the case may be at some scale and some specific factorisation scheme

- Now we are left with

$$d\sigma_{ab}^{real} = d\sigma_{ab}^{real}(\delta_s) + d\sigma_{ab}^{real}(\delta_s, \delta_c) \quad + \quad d\sigma_{ab,fin}^{real}(\delta_s, \delta_c)$$

2-body PS 3-body PS

  - 2-body part which depend explicitly on $\ln\delta_s$ and $\ln\delta_c$

  - 3-body part which when integrated over the phase space using Monte Carlo technique, have an implicit dependence on the same logarithms with opposite signs

  - Physical cross sections are hence independent of these arbitrary cut-off δ_s and δ_c

Virtual Corrections

- For diphoton production including gravity there are no UV singularities—
  - electromagnetic coupling does not receive QCD corrections
  - KK modes couple to SM energy momentum tensor which is a conserved quantity
- Performing the loop integrals the virtual contributions

$$
\begin{aligned}
d\sigma^V &= a_s(\mu_R^2)\, dx_1\, dx_2\, \mathcal{K}(\epsilon, \mu_R^2, s) \\
&\quad \Big\{ C_F \Big[\Big(-\frac{16}{\epsilon^2} + \frac{12}{\epsilon} \Big)\, d\sigma^0_{q\bar q}(\epsilon) + d\sigma^{fin}_{q\bar q} \Big] \Phi_{q\bar q}(x_1, x_2) \\
&\quad + C_A \Big[\Big\{ -\frac{16}{\epsilon^2} + \frac{4}{\epsilon}\frac{1}{C_A}\Big(\frac{11}{3} C_A - \frac{4}{3} n_f T_F \Big) \Big\}\, d\sigma^0_{gg}(\epsilon) + d\sigma^{fin}_{gg} \Big] \Phi_{gg}(x_1, x_2) \Big\}
\end{aligned}
$$

$$
\mathcal{K} = \frac{\Gamma(1+\frac{\epsilon}{2})}{\Gamma(1+\epsilon)} \left(\frac{s}{4\pi\mu_R^2} \right)^{\frac{\epsilon}{2}} \qquad a_s(\mu_R^2) = \frac{\alpha_s(\mu_R^2)}{4\pi}
$$

- SM gluon fusion diagram via quark loop would interfere with the LO gravity mediated diagram, but this is a finite contribution
- SM gluon fusion contribution, though at $\mathcal{O}(\alpha_s^2)$ is comparable to LO for small diphoton invariant mass, but falls of rapidly for larger invariant mass

Real Emission: Leading pole approximation (soft gluon limit)

- Matrix element $$M_3^{soft} = -g_s \mu^{-\epsilon/2} \epsilon_\sigma(p_5) T^a_{ij} \left(\frac{p_2^\sigma}{p_2 . p_5} - \frac{p_1^\sigma}{p_1 . p_5} \right) M_2$$
- Phase Space $(p_5 \longrightarrow 0)$

$$d\Gamma_3^{soft} = d\Gamma_2 \left(\frac{4\pi}{s_{12}} \right)^{-\epsilon/2} \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \frac{1}{2(2\pi)^2} d\mathcal{S}$$

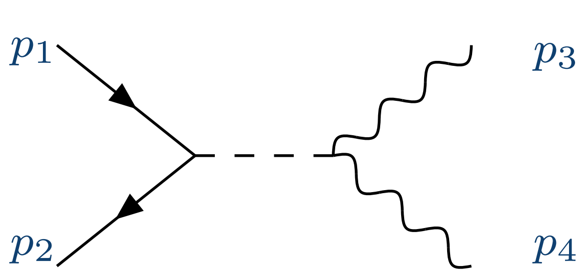
- Performing both E_5 and angular integral over the Eikonal current

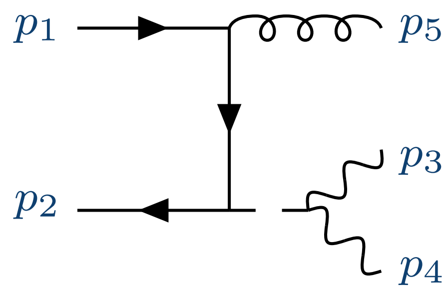
$$d\mathcal{S} \frac{2p_1.p_2}{p_1.p_5\; p_2.p_5} = \frac{8}{\epsilon} \left(\frac{1}{\epsilon} + \ln \delta_s + \frac{\epsilon}{2} \ln^2 \delta_s \right)$$

$$\begin{aligned} d\sigma^{soft} &= a_s(\mu_R^2) dx_1 dx_2 \mathcal{K}(\epsilon, \mu_R^2, s) \left(\frac{16}{\epsilon^2} + \frac{16 \ln \delta_s}{\epsilon} + 8 \ln^2 \delta_s \right) \\ &\quad \Big[C_F d\sigma^0_{q\bar{q}}(x_1, x_2, \epsilon) \Phi_{q\bar{q}}(x_1, x_2) + C_A d\sigma^0_{gg}(x_1, x_2, \epsilon) \Phi_{gg}(x_1, x_2) \Big] \end{aligned}$$

- Pole of order 2 corresponds to soft and collinear gluons and cancels with virtual contributions, while the ϵ^{-1} pole with coefficient $\ln \delta_s$ still remains

Real Emission: Leading pole approximation (Collinear region)





$+$

- Matrix element in the collinear limit $\quad p_t \to 0 \qquad p_1 - p_5 \simeq z p_1$

$$\left| M_3^{col}(q(p_1)\,\bar{q}(p_2) \to \gamma\gamma g) \right|^2 = -\frac{2}{z t_{15}} P_{qq}(z,\epsilon) \left| M_2(q(zp_1)\bar{q}(p_2) \to \gamma\gamma g) \right|^2 g_s^2 \mu_R^{-\epsilon}$$

- Phase space in the collinear limit

$$d\Gamma_3 = d\Gamma_2 \frac{(4\pi)^{-\epsilon/2}}{16\pi^2 \Gamma(1+\epsilon/2)} dz dt_{15} (-(1-z)t_{15})^{\epsilon/2}$$

- Performing dt_{15} integral in the limit $\quad 0 < -t_{15} < \delta_c s_{12}$

$$\begin{aligned} d\sigma_{col} \quad = \quad & a_s(\mu_R^2)\mathcal{K}(\epsilon, \mu_R^2, s_{12}) \;\; d\hat{\sigma}^0_{q\bar{q}}(s_{12}, t_{13}, t_{14}) \;\; f_{\frac{q}{P_1}}(\frac{x_1}{z}) dx_1 \;\; f_{\frac{\bar{q}}{P_2}}(x_2) dx_2 \\ & \frac{1}{\epsilon} P_{qq}(z,\epsilon) \left(\delta_c \frac{1-z}{z}\right)^{\frac{\epsilon}{2}} \frac{dz}{z} \end{aligned}$$

Hard Collinear region

- $z \longrightarrow 1$ is the soft region. Hard region $E_5 > \delta_s \frac{\sqrt{s_{12}}}{2}$ translates to $0 < z < 1 - \delta_s$ for process where soft singularities exist, otherwise there is no restriction on z

$$d\sigma_{HC} = \frac{a_s(\mu_R^2)}{\epsilon} \mathcal{K}(\epsilon, \mu_R^2, s) dx_1 dx_2 \Bigg[d\hat{\sigma}_0^{q\overline{q}}(x_1, x_2, \epsilon)$$

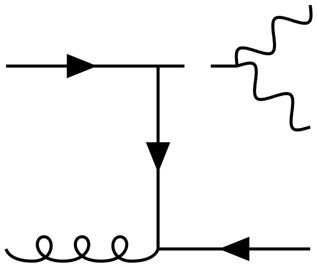
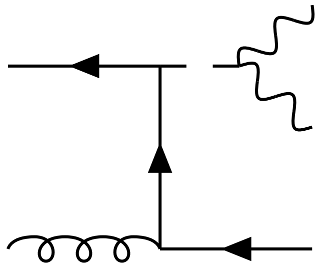
$$\Bigg\{ \int_{x_1}^{1-\delta_s} \frac{dz}{z} \mathcal{H}(z, \epsilon, \delta_c) P_{qq}(z, \epsilon) f_{q_i}(x_1/z) f_{\overline{q}_i}(x_2)$$

$$+ \int_{x_2}^{1-\delta_s} \frac{dz}{z} \mathcal{H}(z, \epsilon, \delta_c) P_{qq}(z, \epsilon) f_{q_i}(x_1) f_{\overline{q}_i}(x_2/z) + x_1 \leftrightarrow x_2 \Bigg\}_{q\overline{q}}$$

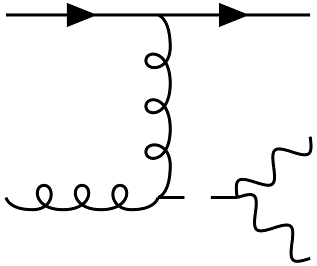
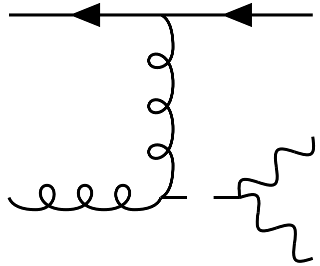
$$\mathcal{H}(z, \epsilon, \delta_c) = \left(\delta_c \frac{1-z}{z} \right)^{\epsilon/2}$$

Hard Collinear region

$$+\, d\hat{\sigma}_0^{q\overline{q}} \int_{x_2}^{1} \frac{dz}{z} \mathcal{H}(z,\epsilon,\delta_c) \Big\{ P_{qg}(z,\epsilon) f_{q_i}(x_1) f_g(x_2/z) + P_{qg}(z,\epsilon) f_{\overline{q}_i}(x_1) f_g(x_2/z) + x_1 \leftrightarrow x_2 \Big\}_{qg}$$

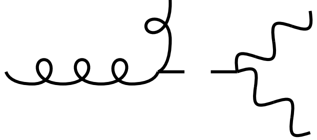
$$+\, d\hat{\sigma}_0^{gg} \int_{x_1}^{1} \frac{dz}{z} \mathcal{H}(z,\epsilon,\delta_c) \Big\{ P_{gq}(z,\epsilon) f_{q_i}(x_1/z) f_g(x_2) + P_{gq}(z,\epsilon) f_{\overline{q}_i}(x_1/z) f_g(x_2) + x_1 \leftrightarrow x_2 \Big\}_{qg}$$

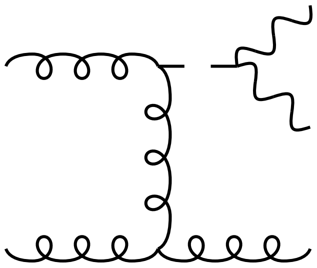
 

- Particle emitted in the final state is a fermion— no soft singularities

Hard Collinear region

$$+ d\hat{\sigma}_0^{gg}(x_1, x_2, \epsilon) \int_{x_1}^{1-\delta_s} \frac{dz}{z} \mathcal{H}(z, \epsilon, \delta_c) P_{gg}(z, \epsilon) \left\{ f_g(x_1/z) f_g(x_2) + x_1 \leftrightarrow x_2 \right\}_{gg} \Bigg]$$





Renormalised parton distributions $\overline{MS}$ scheme

$$f_q(x) = f_q(x,\mu_F) - \frac{a_s(\mu_R^2)}{\epsilon} \frac{\Gamma(1+\epsilon/2)}{\Gamma(1+\epsilon)} \left(\frac{\mu_F^2}{4\pi\mu_R^2}\right)^{\frac{\epsilon}{2}} \int_x^1 \frac{dz}{z} \Big[P_{qq}(z) f_q(\frac{x}{z}) + P_{qg}(z) f_g(\frac{x}{z})\Big]$$

$$f_g(x) = f_g(x,\mu_F) - \frac{a_s(\mu_R^2)}{\epsilon} \mathcal{K}(\epsilon,\mu_R^2,s) \left(\frac{\mu_F^2}{s}\right)^{\frac{\epsilon}{2}} \int_x^1 \frac{dz}{z} \Big[P_{gg}(z) f_g(x/z) + P_{gq}(z)\big(f_q(x/z) + f_{\overline{q}}(x/z)\big)\Big]$$

• Counter terms to cancel the collinear singularities are obtained by substituting the renormalised PDFs in LO cross section— mass factorisation

$$\begin{aligned} d\sigma_0 &= dx_1\, dx_2\, d\hat{\sigma}_0^{q\overline{q}}(x_1,x_2,\epsilon) \sum_i \Big[f_{q_i}(x_1) f_{\overline{q}_i}(x_2) + f_{\overline{q}_i}(x_1) f_{q_i}(x_2)\Big] \\ &+ dx_1\, dx_2\, d\hat{\sigma}_0^{gg}(x_1,x_2,\epsilon) f_g(x_1) f_g(x_2) \end{aligned}$$

Cancellation of collinear singularities

- $d\sigma^{HC}$ for $d\hat{\sigma}_0^{q\overline{q}}(x_1, x_2, \epsilon)$ hard process to $\mathcal{O}(a_s)$

$$\frac{1}{\epsilon}\mathcal{K}(\epsilon, \mu_R^2, s) f_q(x_1) \int_{x_2}^{1-\delta_s} \frac{dz}{z} \mathcal{H}(z, \epsilon, \delta_c) P_{qq}(z, \epsilon) f_{\overline{q}}(x_2/z)$$

- Corresponding counter term

$$-\frac{1}{\epsilon}\mathcal{K}(\epsilon, \mu_R^2, s) f_q(x_1, \mu_F) \left(\frac{\mu_F^2}{s}\right)^{\frac{\epsilon}{2}} \int_{x_2}^{1} P_{qq}(z) f_{\overline{q}}(x_2/z)$$

- Cancellation of collinear singularities in the hard collinear region $d\sigma^{HC}$ are not complete when the counter terms $d\sigma^{CT}$ is added— as the phase space is separated into soft and hard regions

$$\frac{1}{2} f_q(x_1, \mu_F) \int_{x_2}^{1-\delta_s} \frac{dz}{z} \left\{ P_{qq}(z) \ln\left[\delta_c \frac{1-z}{z} \frac{s}{\mu_F^2}\right] - P'_{qq}(z) \right\} f_{\overline{q}}(x_2/z)$$

$$+\left[-\frac{1}{\epsilon} + \frac{1}{2}\ln\left(\frac{s}{\mu_F^2}\right)\right] f_q(x_1, \mu_F) \int_{1-\delta_s}^{1} \frac{dz}{z} P_{qq}(z) f_{\overline{q}}(x_2/z)$$

Cancellation of collinear singularities

$$
\begin{aligned}
d\sigma^{HC+CT} &= a_s(\mu_R^2)\, dx_1\, dx_2\, \mathcal{K}(\epsilon, \mu_R^2, s) \\
& d\hat{\sigma}^0_{ab}\Big\{\frac{1}{2}\Big(f_a(x_1,\mu_F)\, \widetilde{f}_b(x_2,\mu_F) + \widetilde{f}_a(x_1,\mu_F)\, f_b(x_2,\mu_F)\Big) \\
& +2\Big(-\frac{1}{\epsilon}+\frac{1}{2}\ln\frac{s}{\mu_F^2}\Big)\, A_{a\to b+c}\, f_a(x_1,\mu_F)\, f_b(x_2,\mu_F) + x_1 \leftrightarrow x_2\Big\} \\
\widetilde{f}_q(x_1,\mu_F) &= \int_x^{1-\delta_s}\frac{dz}{z}\widetilde{P}_{qq}(z) f_q\Big(\frac{x}{z},\mu_F\Big) + \int_x^1 \frac{dz}{z}\widetilde{P}_{qg}(z) f_g(x/z,\mu_F) \\
\widetilde{f}_g(x_1,\mu_F) &= \int_x^1 \frac{dz}{z}\widetilde{P}_{gq}(z) f_q\Big(\frac{x}{z},\mu_F\Big) + \int_x^{1-\delta_s} \frac{dz}{z}\widetilde{P}_{gg}(z) f_g(x/z,\mu_F)
\end{aligned}
$$

$$
A_{a\to b+c} \equiv \int_{1-\delta_s}^1 \frac{dz}{z} P_{ab}(z) = \begin{pmatrix} \underbrace{8C_F\ln\delta_s} + 6C_F & 0 \\ 0 & \frac{22}{3}C_A - \frac{8}{3}n_f T_F + \underbrace{8C_A\ln\delta_s} \end{pmatrix}
$$

- All $+$ive powers of $\delta_s \to 0$

Cancellation of collinear singularities

- Now all poles of soft and collinear origin automatically cancel and is left with a $finite$ 2-body process explicitly dependent on $\ln\delta_s$ and $\ln\delta_c$

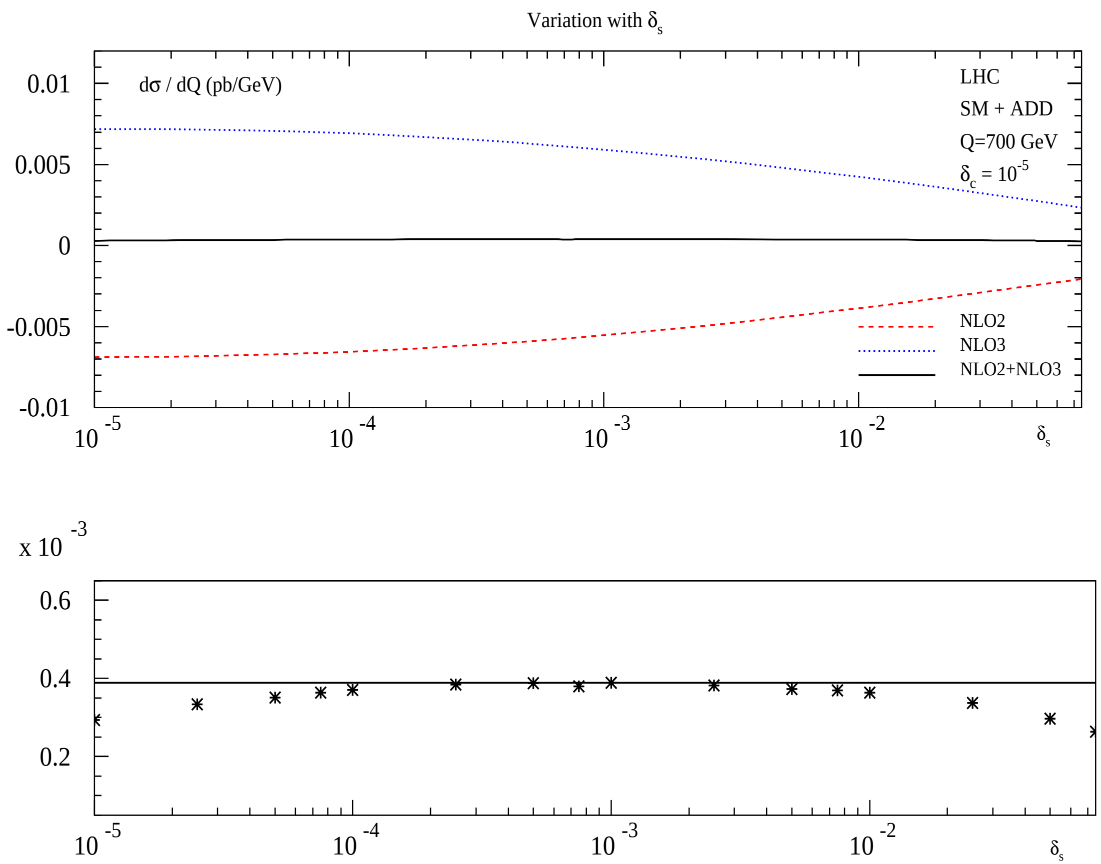
$$d\sigma^{2-body}(\delta_s,\delta_c) \;=\; d\sigma^{virt} + d\sigma^{real}(\delta_s) + d\sigma^{HC+CT}(\delta_s,\delta_c)$$

- This when added to the finite hard non-collinear 3-body contribution and on performing the phase space integration using a Mote Carlo techniques have implicit dependence on the same logarithms with opposite signs

$$d\sigma \;=\; d\sigma^{2-body}(\delta_s,\delta_c) + d\sigma^{3-body}(\delta_s,\delta_c)$$

- For a reasonable range where δ_s and δ_c are small, the results are stable, providing a check on the calculation
- The combined analytic and Monte Carlo method is flexible enough to accommodate various experimental cuts and compute different observables that are infrared and collinear safe

Stability plot for $d\sigma/dQ$ (SM+ADD)



- Numerical results least sensitive to slicing parameters over a wide range
- Choose a particular value for $\delta_{s,c}$ (stable region) for numerical predictions

Di-photon signal

- Prompt photons:
  - Direct: Both photons originating from the hard partonic interaction
  - Fragmentation: At least one photon produced in the hadronisation of a parton
- Fragmentation photon would be accompanied by hadronic activity in its vicinity
- Final state quark-photon collinear singularity appears in the calculation of the subprocess $gq \to q\gamma\gamma$.
- These singularities can be factorised and absorbed into the fragmentation functions $D_{\gamma/a}(z, \mu_F)$ where $a = q, \overline{q}, g$, to all orders in α_s — additional non perturbative input

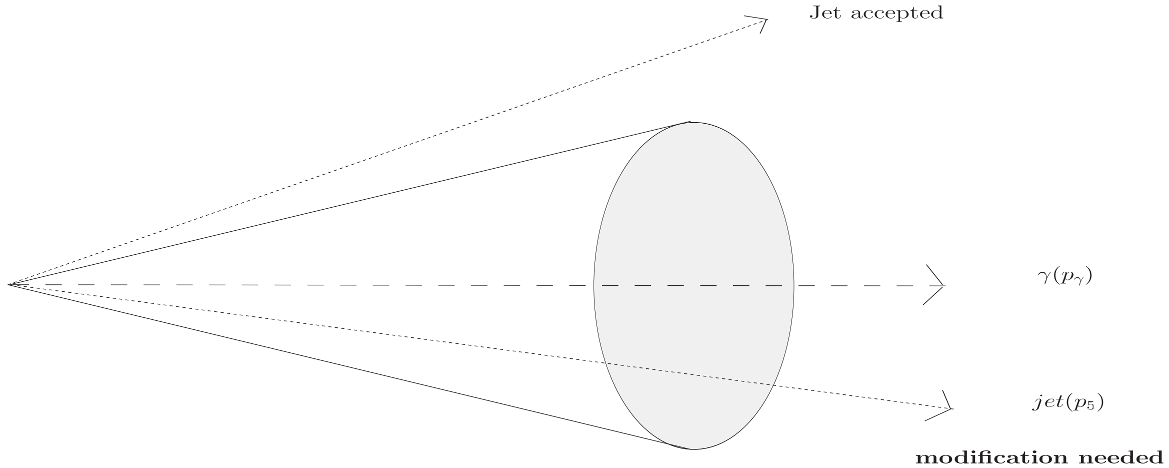
Binoth *et.al* arXiv:hep-ph/9911340

- We adopt an alternate smooth cone isolation criterion proposed by Frixione which ensures that the fragmentation contribution are suppressed with out affecting the cancellation of any of the singularities discussed earlier.

Frixione arXiv:hep-ph/9801442

Frixione's algorithm for the isolation of photons

- Method to define an isolated photon is to draw a circle of radius r_0 in the (η, ϕ) plane, centered on the photon candidate



- Demanding no hadronic activity in the region $r < r_0$ would not only remove the fragmentation contribution but also gluons from that region of *phase space*— event not IR safe

- Fragmentation mechanism is a collinear phenomenon, to eliminate its contribution— sufficient to veto only collinear configurations

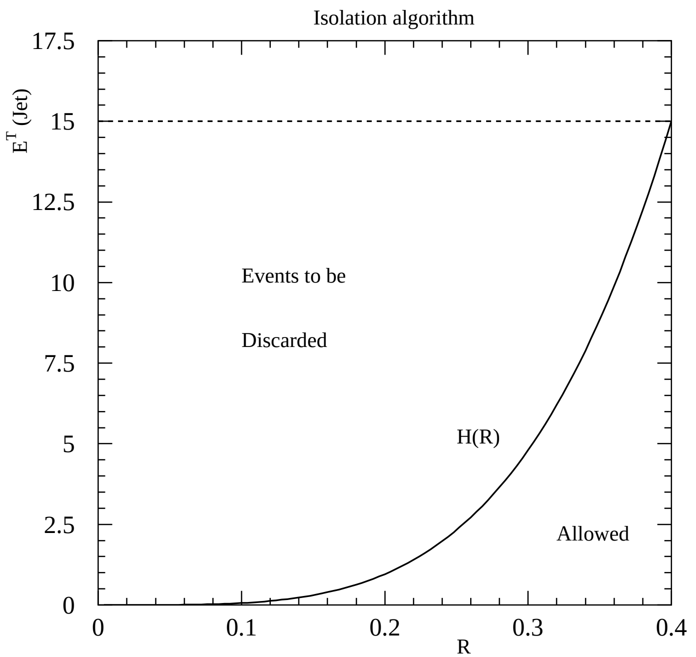
Smooth cone isolation prescription

• Define a continuous set of circles with $r < r_0$ and demand total transverse energy of hadronic activity permitted inside r, $E_T(r)$ decreases to zero as $r \to 0$

• $\sum_i E_{T,i} \leq E_T^{iso} \left(\frac{1-\cos(r)}{1-\cos(r_0)} \right)^n$

• Energy of parton emitted exactly collinear to the photon must vanish

• Contribution of fragmentation is restricted to $D_{\frac{\gamma}{q,g}}(z)\Big|_{z=1} = 0$

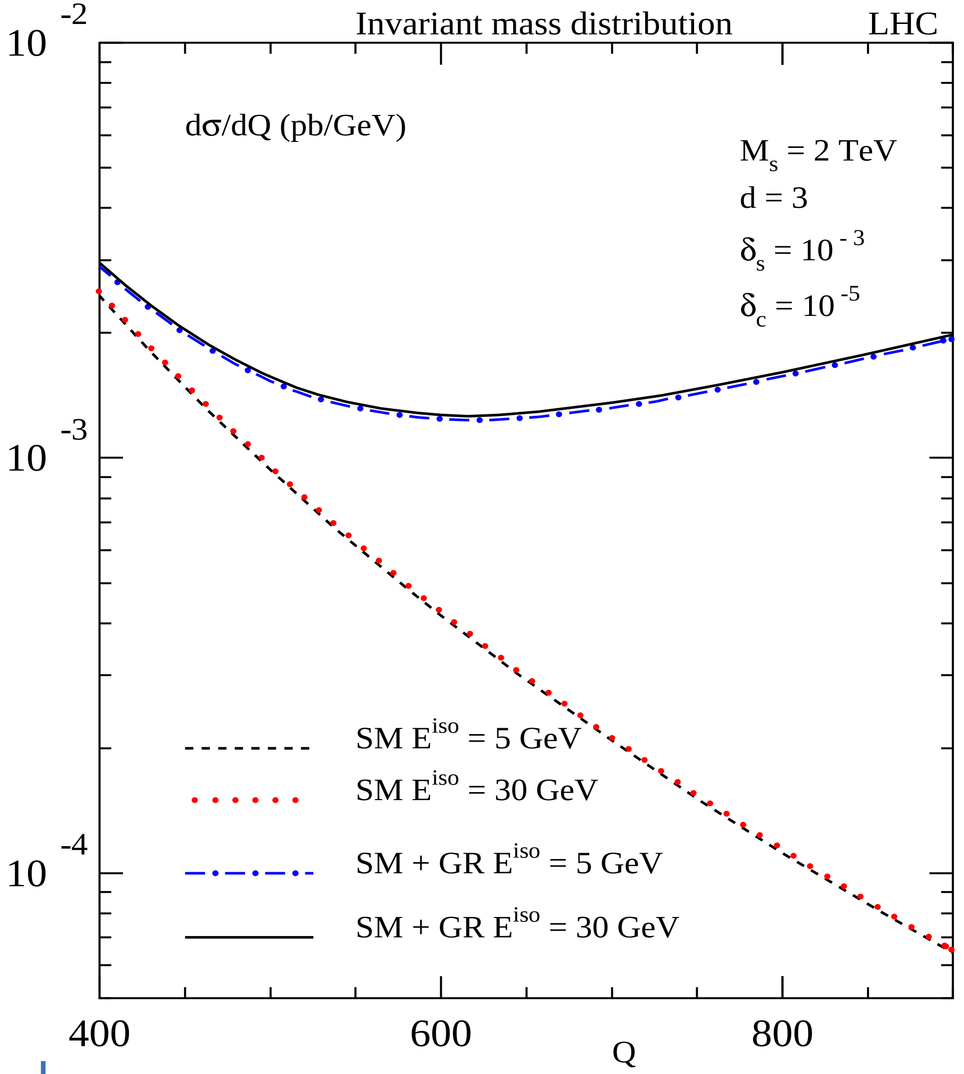


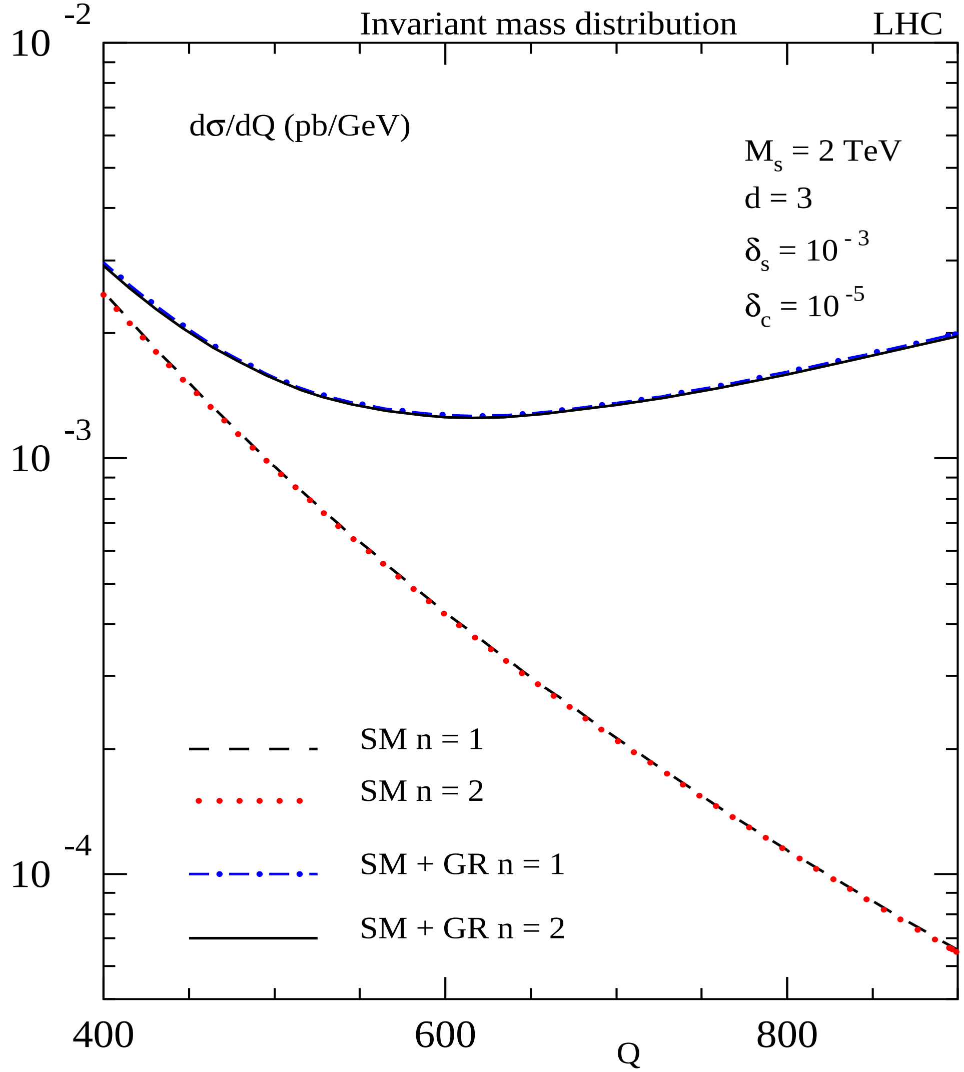
• No region of phase space is forbidden to radiation and at the same time has the virtue of entirely suppressing the poorly known non perturbative fragmentation contribution

Dependence of photon isolation criteria E_T^{iso} and n

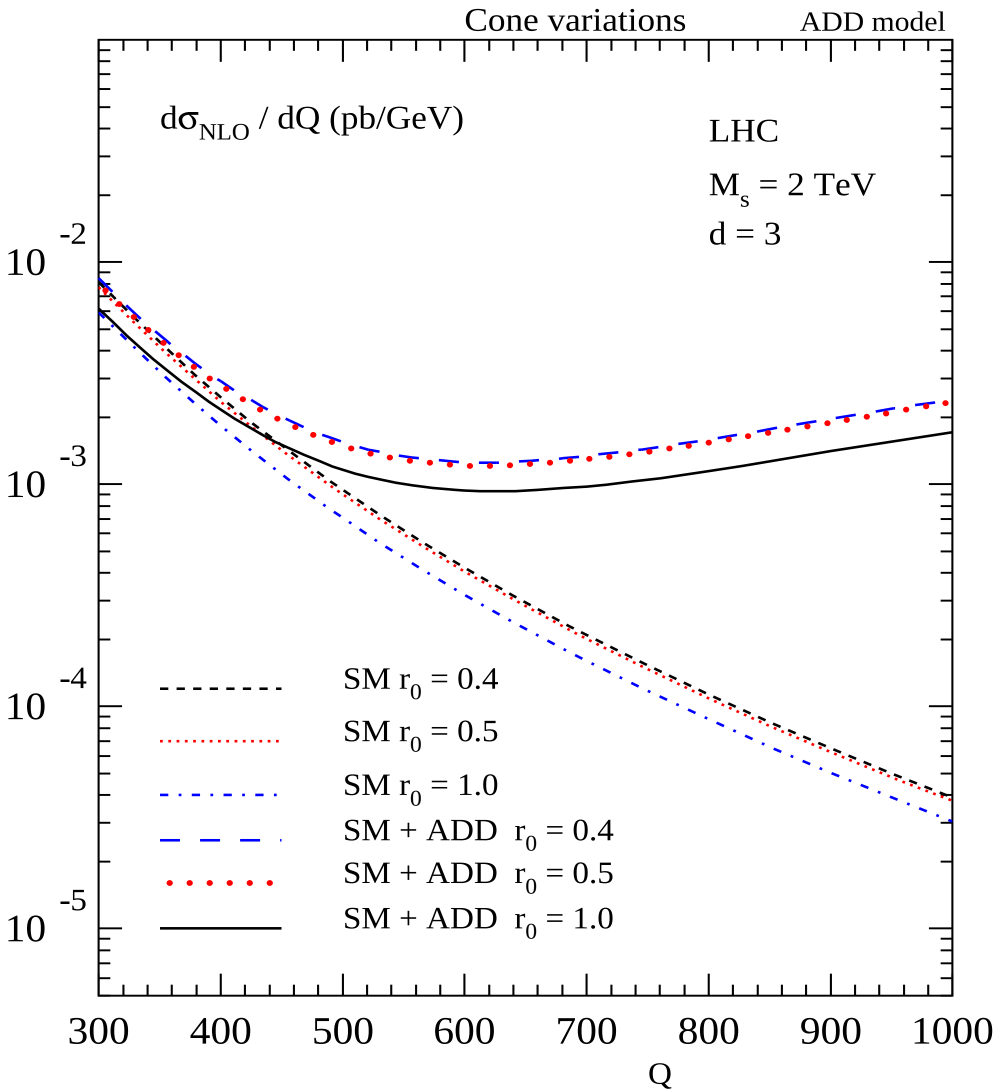
- ADD

Default choice $E_T^{iso} = 15$ GeV, $n = 2$, $r_0 = 0.4$





Effects of varying the cone size r_0



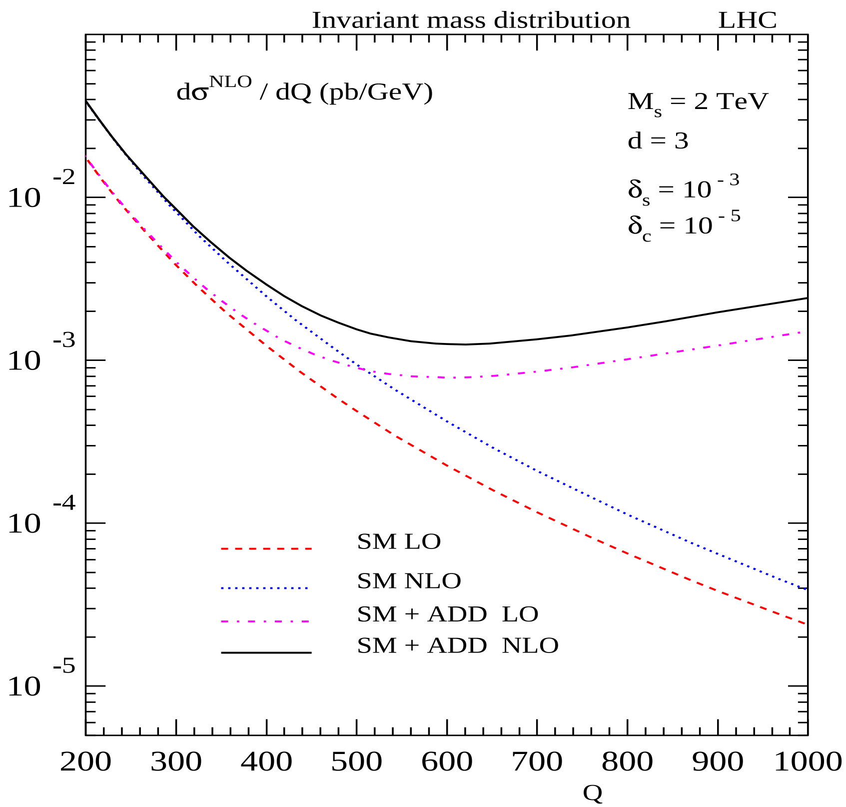
Numerical Results

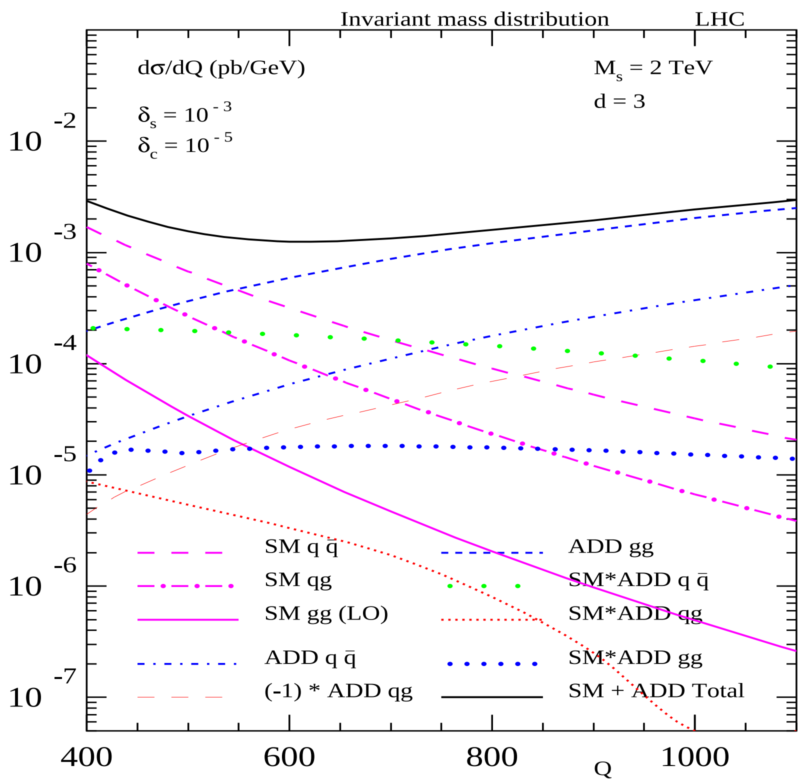
- Phase space slicing parameters $\delta_s = 10^{-3}$ and $\delta_c = 10^{-5}$
- Photon isolation criteria $E_T^{iso} = 15$ GeV, $n = 2$, $r_0 = 0.4$
- Parton Distribution Functions:
  - LO CTEQ6L
  - NLO CTEQ6M

$n_f = 5$ light quark flavours and $\mu_F = \mu_R = Q$

- ADD parameters $M_s = 2$ TeV, $d = 3$
- RS parameters $M_1 = 1.5$ TeV, $c_0 = 0.01$
- Kinematical cuts: (ATLAS & CMS)
  - $p_T^\gamma > 40(25)$ GeV for harder (softer) photons
  - $|y_\gamma| < 2.5$ for each photon
  - $r_{\gamma\gamma} = 0.4$ minimum separation between two photons in (η, ϕ) plain

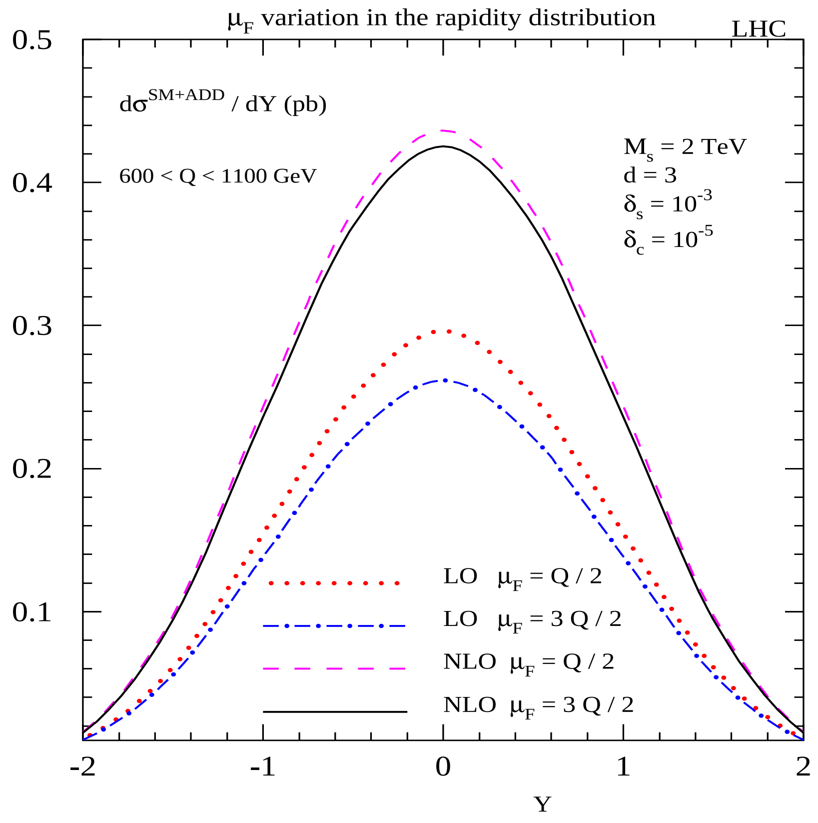
Invariant mass distribution of the diphoton $d\sigma/dQ$ (ADD)





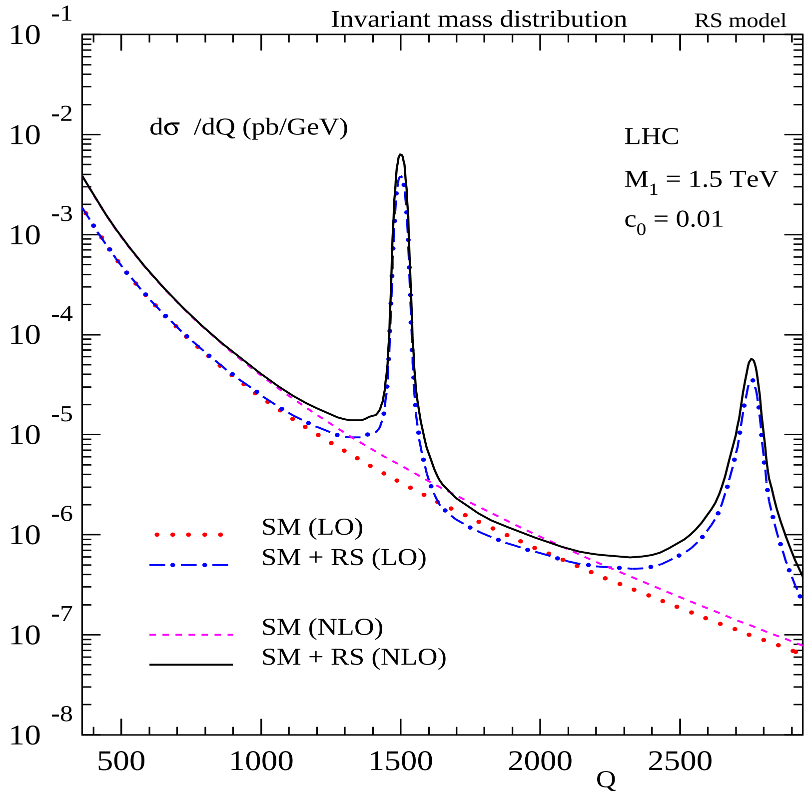
- SM gg-fusion process through quark loop ($\mathcal{O}(\alpha_s^2)$), is comparable to LO in the lower invariant mass Q region but falls of rapidly in the region of interest to large extra dim models

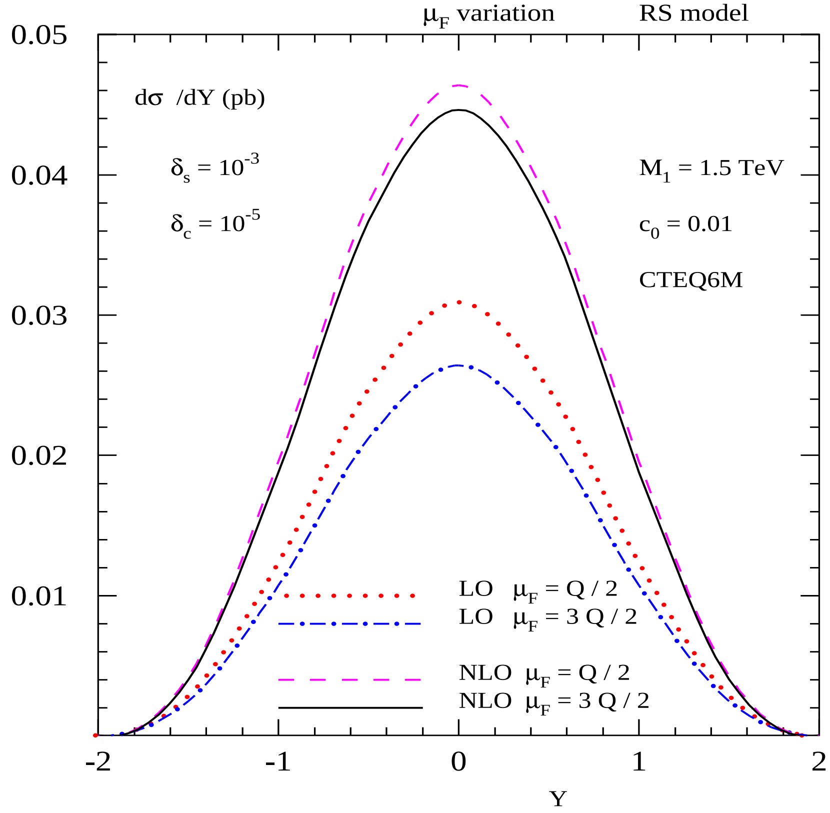
Factoriasation scale dependence of $d\sigma/dY$



- NLO results show significant improvement on the factorisation scale uncertainty entering thorough the PDFs at LO

Invariant mass distribution of the diphoton $d\sigma/dQ$ (RS)





Summary

- Asymptotic freedom, factorisation and evolution the instruments needed to analyse QCD processes at colliders

- Perturbative QCD is an essential and established part of the tool kit and has attained sufficient precision to search for new physics

- Di-photon signal to NLO using the semi analytical phase space slicing method is flexile enough to accommodate various experimental cuts and compute different observables that are infrared and collinear safe
- Quantitative impact of the QCD corrections for searches of extra dimension at hadron colliders

Natural question

- **Is this leading order result stable in the perturbation theory?**
- **Why should we ask this question at all here?**
- **Because we are dealing with partons such as quark and gluons at the initial state which are sensitive to Factorisation scale even at LO**

$$d\sigma^{PP}\left(x,Q^2\right)=\sum_{ab}\int_x^1\frac{dz}{z}\Phi_{ab}^{(0)}\left(z,Q^2,\mu_F^2\right)\sigma_{ab}^{(0)}\left(\frac{x}{z},Q^2,M_S^2\right)+\cdots$$

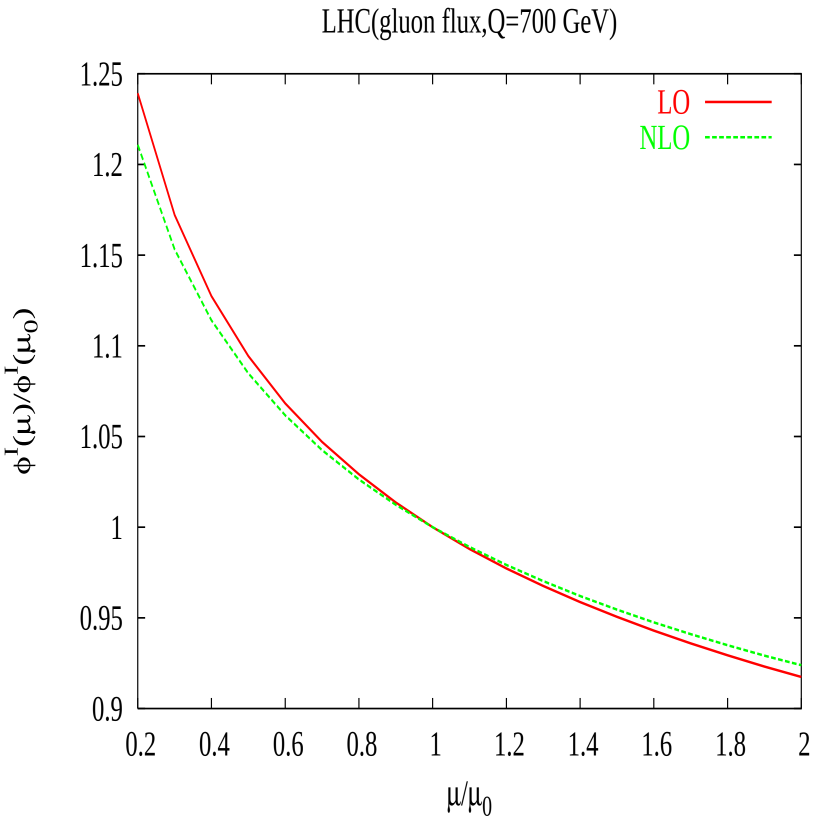
Leading Partonic cross section is "independent" of μ_F

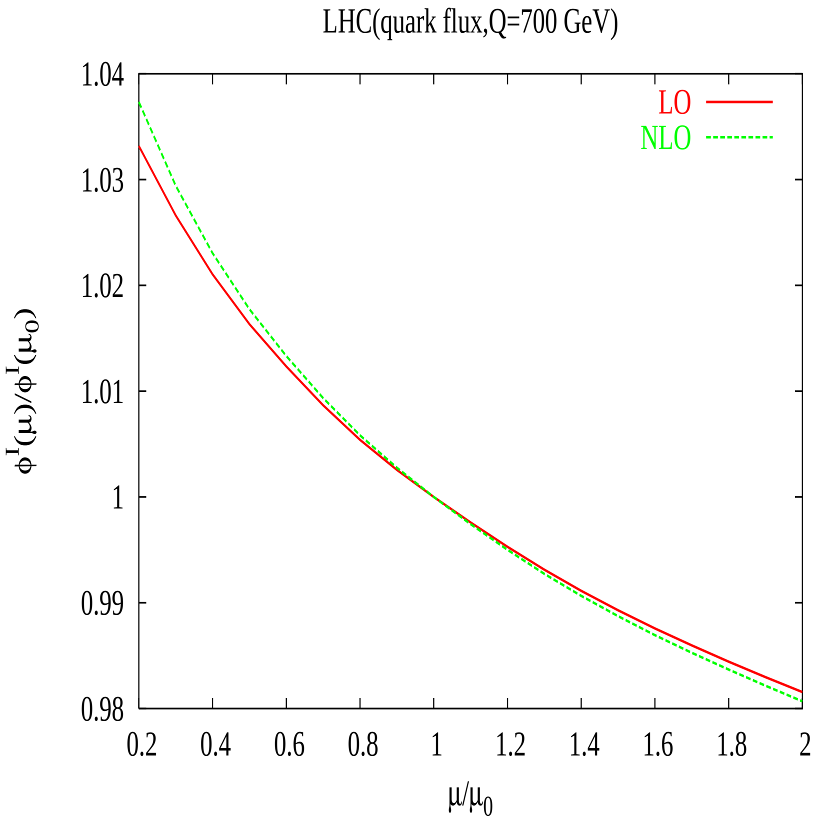
- **Uncertainty can come from Factorisation scale μ_F through the LO flux** $\Phi_{ab}^{(0)}\left(z,\mu_F\right)$
- **How serious is it?**

Scale Variation of Flux at LHC

$$\Phi^I_{ab}(\tau, \mu_F) \quad = \quad \int_\tau^1 \frac{dz}{z} f_a\left(z, \mu_F\right) f_b\left(\frac{\tau}{z}, \mu_F\right) \qquad I = LO, \;\; NLO$$

$$\mu_0 = 0.7 \text{ TeV} \qquad\qquad \tau = \frac{Q^2}{S} \qquad\qquad \sqrt{S} = 14 \text{ TeV}$$

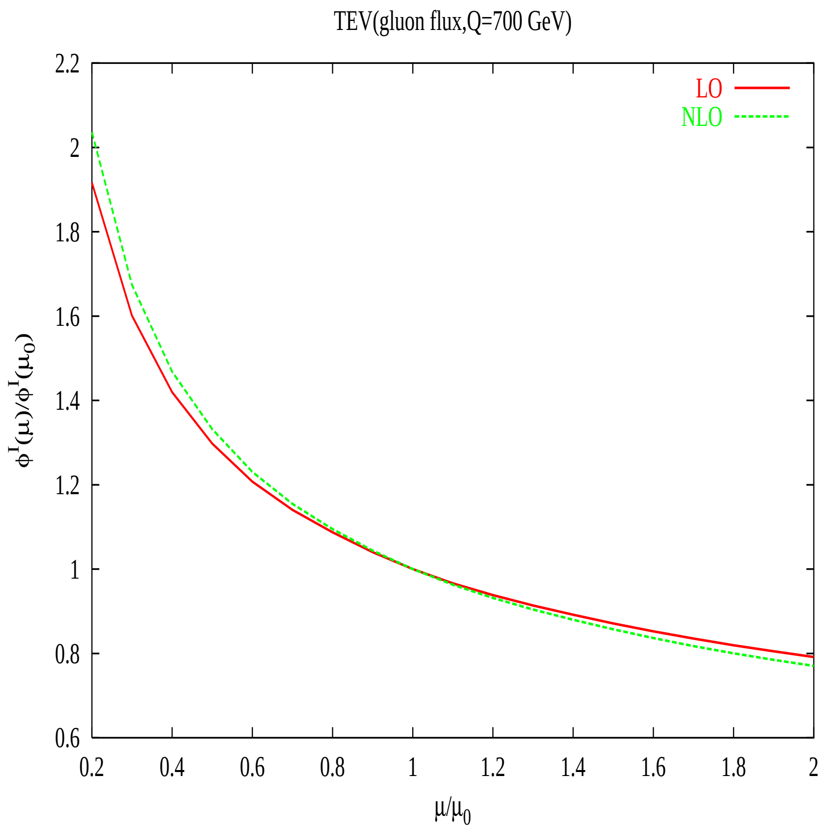


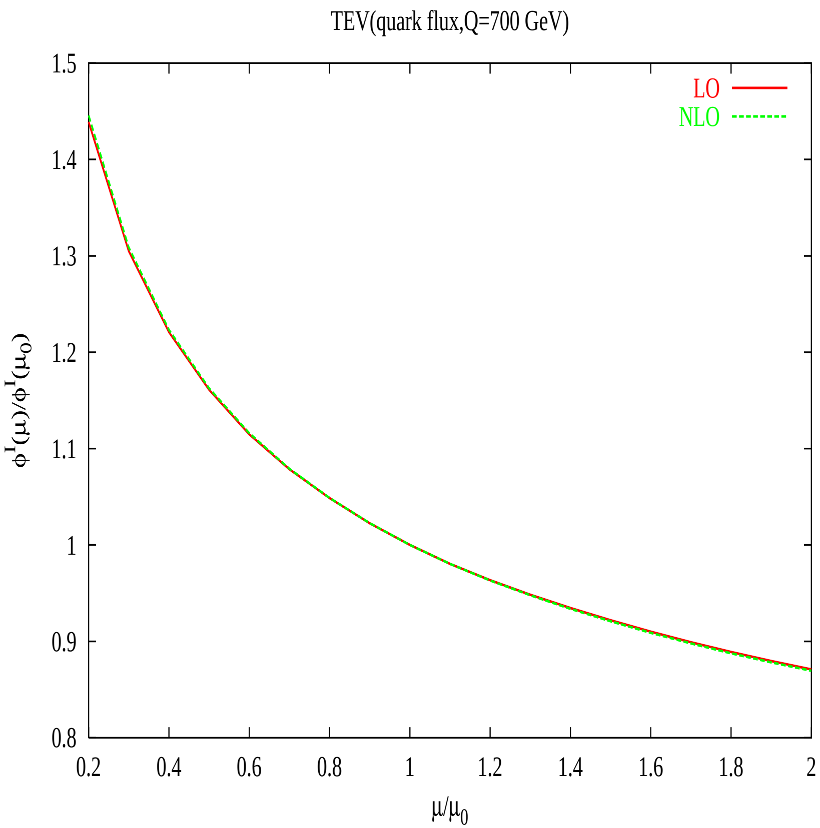


Scale Variation of Flux at Tevatron

$$\Phi^I_{ab}(\tau,\mu_F) \quad = \quad \int_\tau^1 \frac{dz}{z} f_a\left(z,\mu_F\right) f_b\left(\frac{\tau}{z},\mu_F\right) \qquad I = LO,\;\; NLO$$

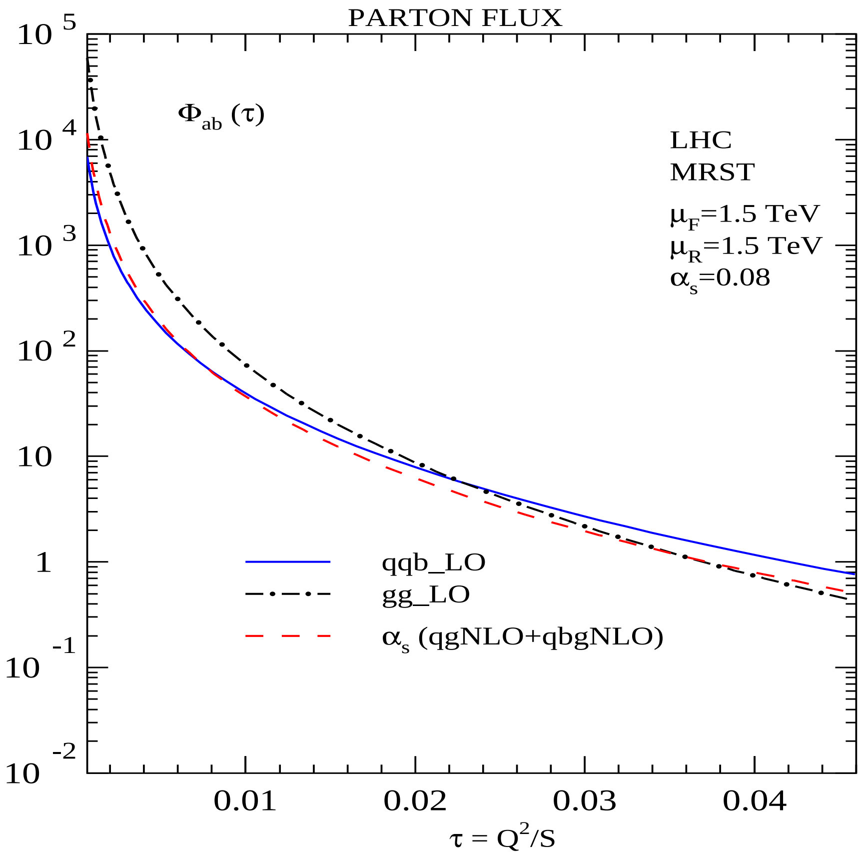
$$\mu_0 = 0.7 \text{ TeV} \qquad\qquad \tau = \frac{Q^2}{S} \qquad\qquad \sqrt{S} = 1.96 \text{ TeV}$$

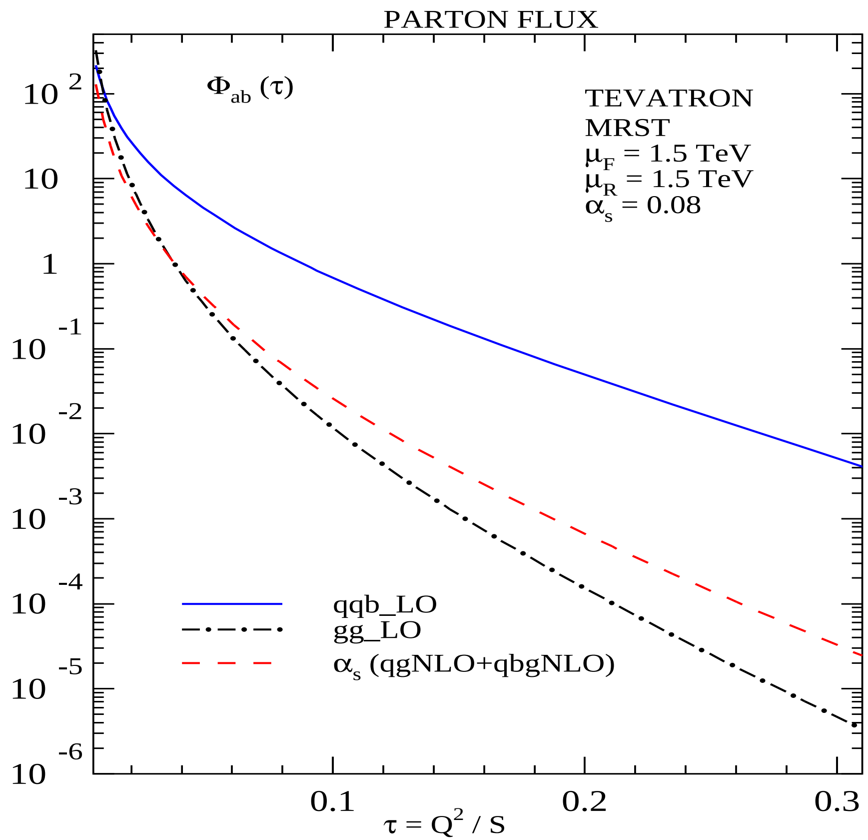




Flux at LHC and Tevatron

$$\Phi_{ab}(\tau,\mu_F) \quad = \quad \int_\tau^1 \frac{dz}{z} f_a\left(z,\mu_F\right) f_b\left(\frac{\tau}{z},\mu_F\right) \qquad\qquad \tau = \frac{Q^2}{S}$$





- **Gluon flux dominates at LHC**
- $q\ \bar{q}$ **flux is largest at Tevatron**