RF Accelerators

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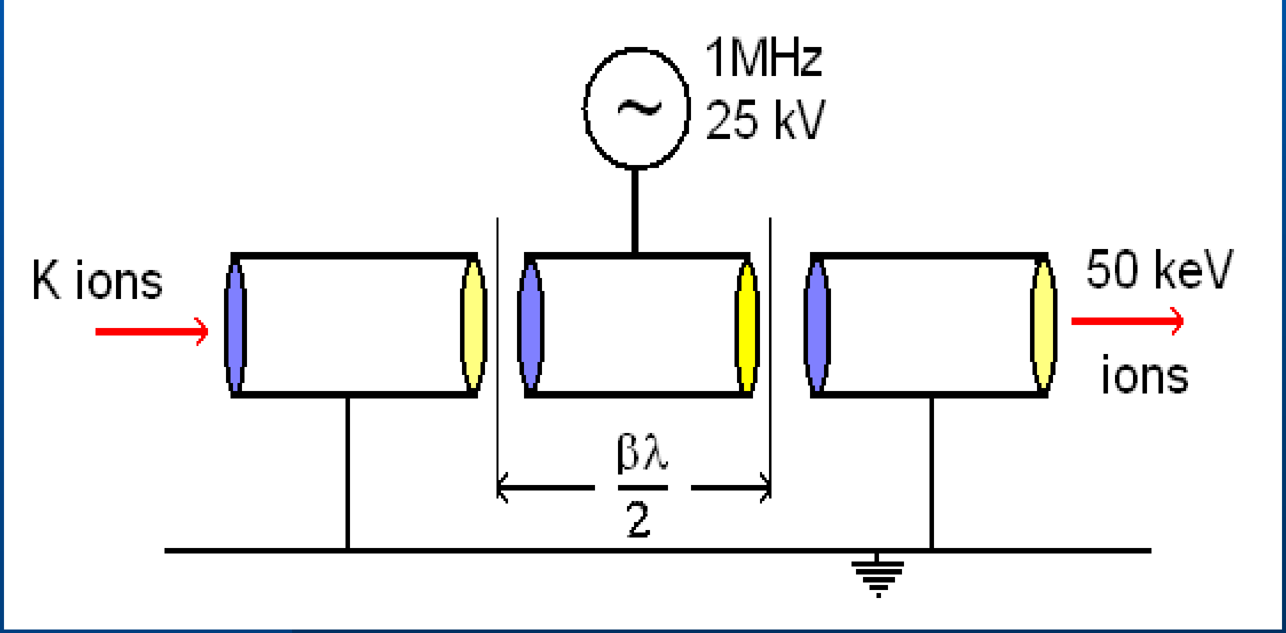
Successful Attempts to fool the electromagnetic waves

The principle described in paper by G. Ising (1924)

R. Wideroe(1928):

First successful acceleration of K ion to 50 keV

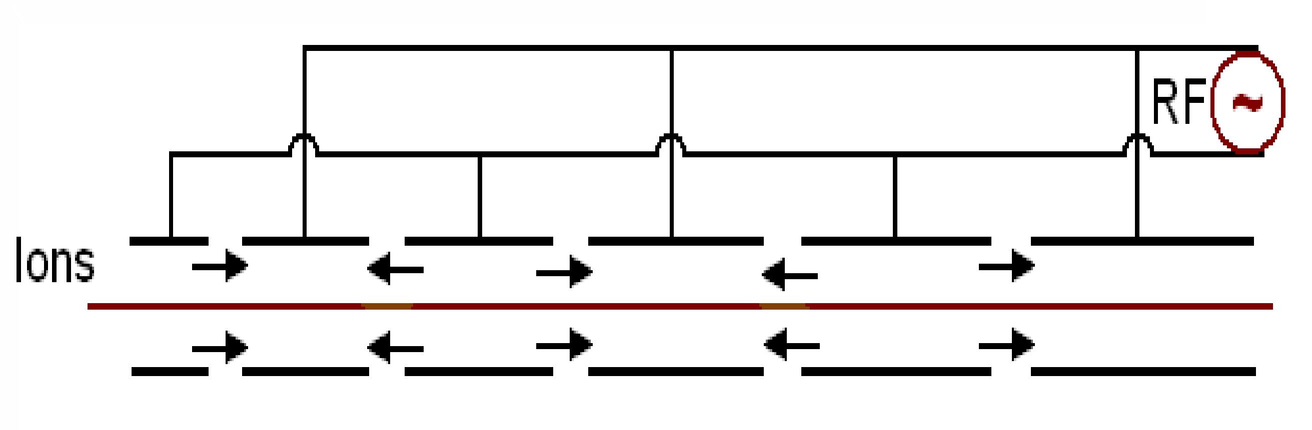


$$d = \frac{v}{2f} = \frac{\beta\lambda}{2}$$

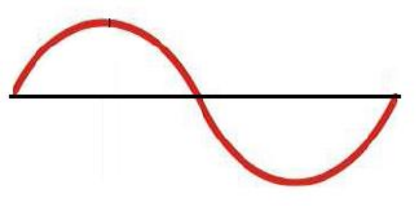
Wideroe linac

E. O. Lawrence and D. Sloan:

Build a Wideroe type $\beta\lambda / 2$ linac with 30 drift tubes, 42 kV@ 7 MHz, mercury ions ~ 1.26 MeV.



$$d = \frac{v}{2f} = \frac{\beta\lambda}{2}$$

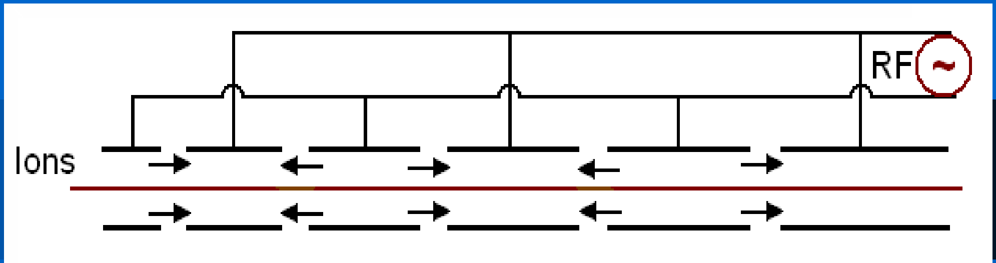


Used for heavy ions $\beta < 0.03$.
$f < 100$ MHz for practical gaps

length of the drift tubes should increase as energy increases

- For higher energies, high RF frequency is needed
- No continuous acceleration of beams of particles

Wideroe Linac



$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

Problem

- At higher frequencies gap is almost capacitive
- Radiate a large amount of energy

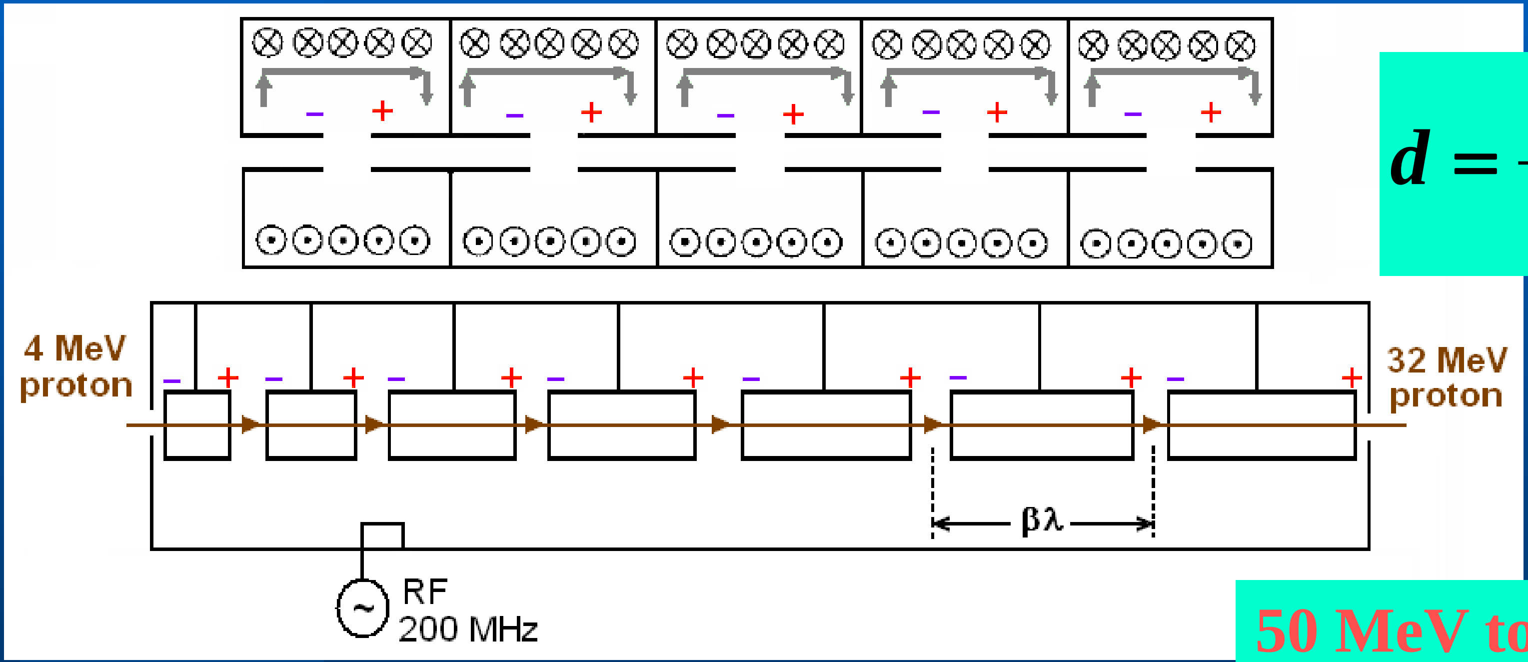
The displacement current flowing $I = \omega C V$

- Enclose the gap existing between drift tubes in a cavity
- Store electromagnetic energy in the form of a magnetic field

Alvarez linac

L. Alvarez (1945) : 32 MeV proton drift tube linac @ 200 MHz

Particles traverse each drift tube in one full RF period



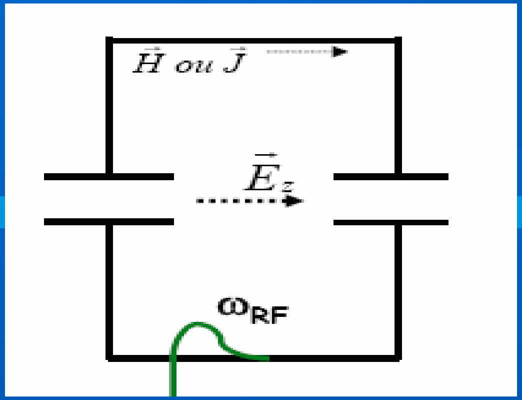
$$d = \frac{v}{f} = \beta\lambda$$

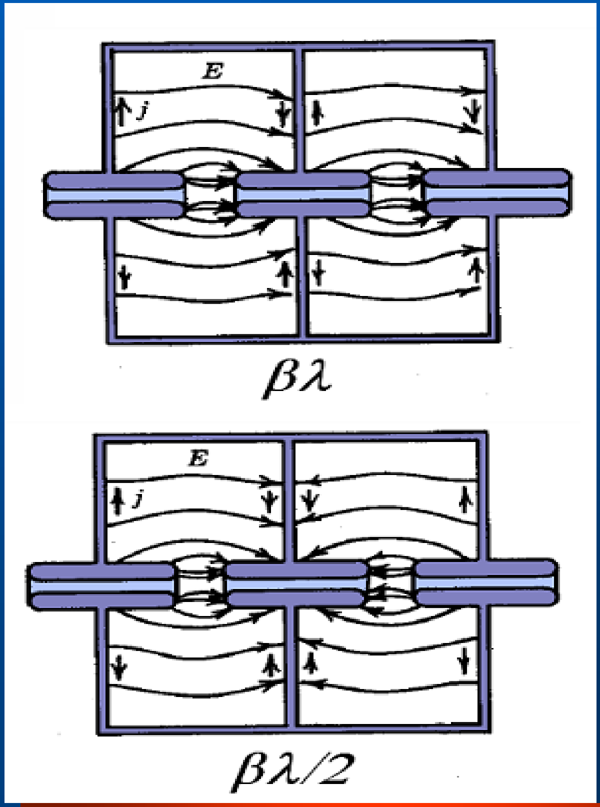
50 MeV to 200 MeV.

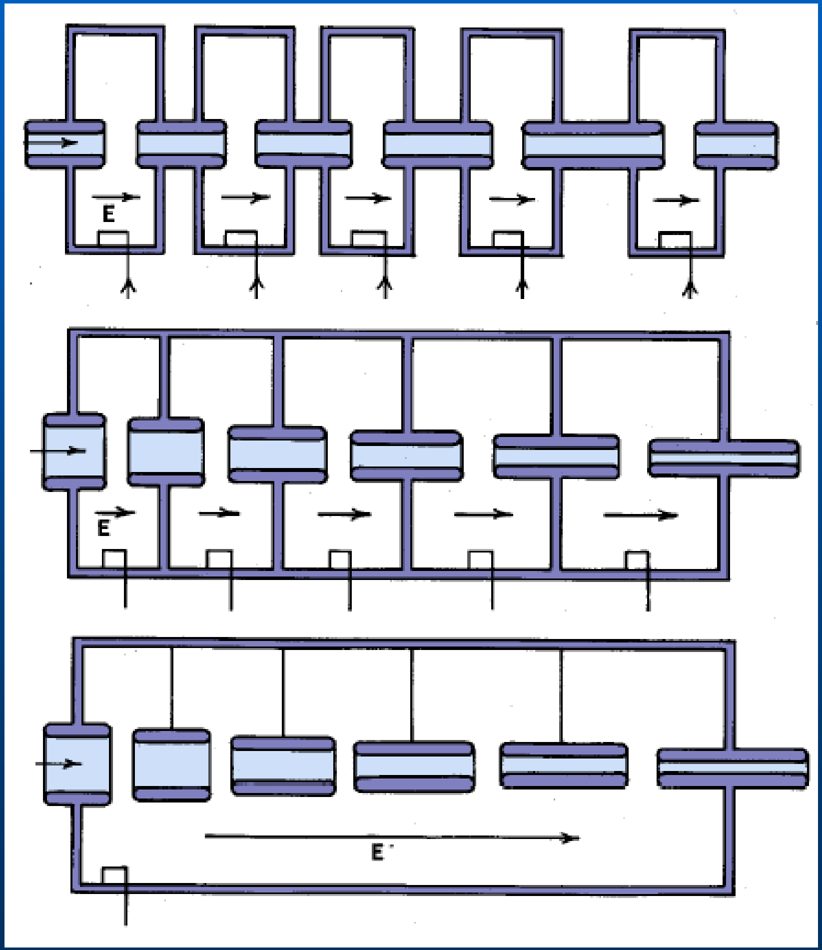
- This linac is a standing wave structure in 0 or 2π mode
- Very efficient for $0.05 < \beta < 0.3$ used for protons & heavy ions

Cavity

We need to know more about cavity







Traveling & standing wave structures

$$d = \frac{v}{2f} = \frac{\beta\lambda}{2}$$

- Higher frequencies are desired for ultra relativistic particles
- At β~1 *d*, the length of drift tube will remain constant.
- EM wave in free space which also moves with velocity c ???

  NO!!!!!
- Field is normal to wave direction

There are two Solutions

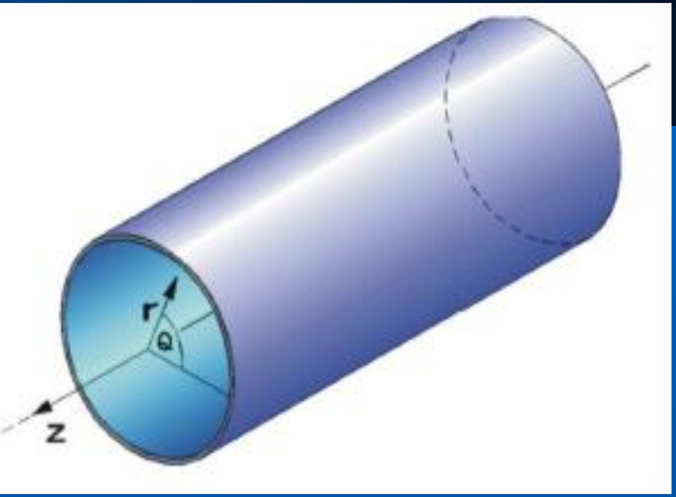
- Keep bending beam back into the wave

  (principle of inverse free electron laser accelerator)
- Reflect the wave back into the wave

  (principle of all linacs using waveguides).

Traveling & standing wave structures

Cross section of most linear accelerator are circular



Consider TM waves in a cylindrical waveguide (no θ dependence).

The wave equation is

$$\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E_z}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2} = 0$$

$$E_z = Z(z)R(r)T(t)$$

$$T(t) \propto \exp(j\omega t)$$
$$Z(z) \propto \exp(-jkz)$$

$$\frac{d^2 R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + k_r^2 R = 0$$

$$k_r^2 = \frac{\omega^2}{c^2} - k^2$$

Bessel equation of zero order

$$R(r) = AJ_0(k_r r)$$

Brillouin diagram

$$R(r) = A J_0(k_r r)$$

$$\frac{\omega^2}{c^2} = k_r^2 + k^2$$

At $r = a$, $J_0(k_r r) = 0$

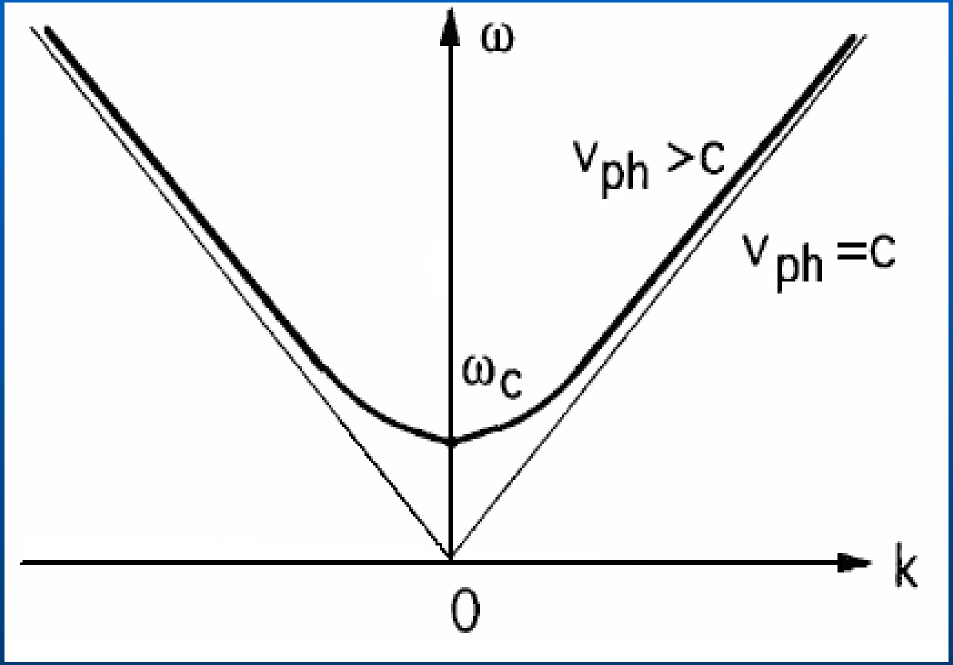
Many roots, many modes.
The first root is mode TM_{01}.

$$k_r a = 2.405$$

Dimension of the guide fixes the value of k_r

Phase velocity $v_{ph} = \omega / k$.

Group velocity $v_g = d\omega / dk$



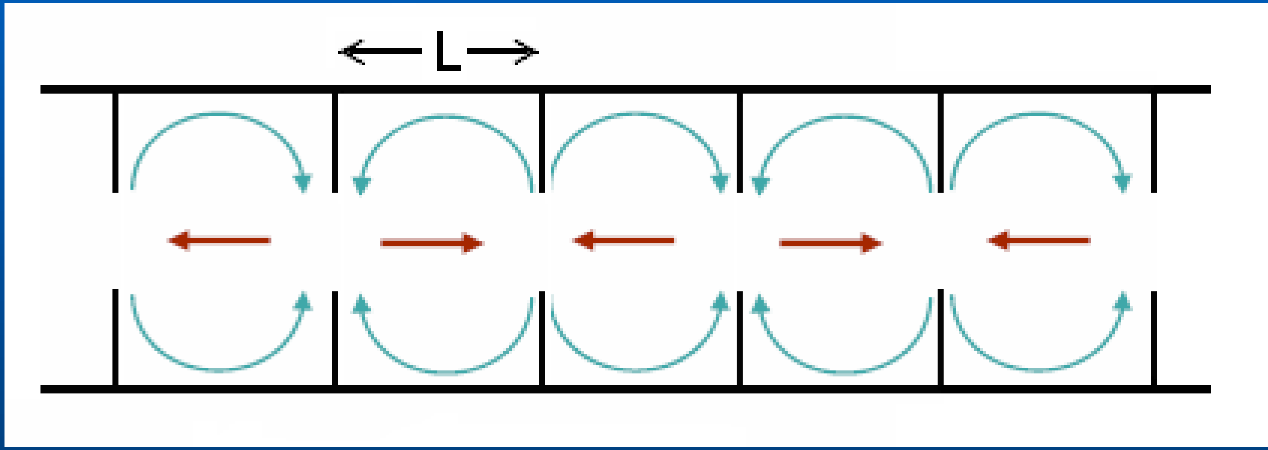
- In uniform wave guide $v_{ph} > c$.
- Unsuitable for acceleration.
- Lowest frequency is ω_c,

$v_g = 0 \quad v_{ph} = \infty$

Loaded waveguide

For acceleration of particle synchronism is must.
Phase velocity v_{ph} = Particle velocity v_p. ($v_{ph} > c$)

Slow down the waves by 'loading' the cavity

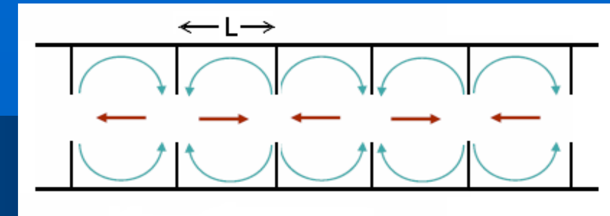


$$\frac{\omega^2}{c^2} = k_r^2 + \left[k + \frac{2\pi n}{L}\right]^2$$

$$v_{ph} = \frac{\omega}{k + (2\pi n / L)} = \frac{\omega}{k_n}$$

Each *n* has a traveling wave

Brillouin diagram of Loaded Cavity



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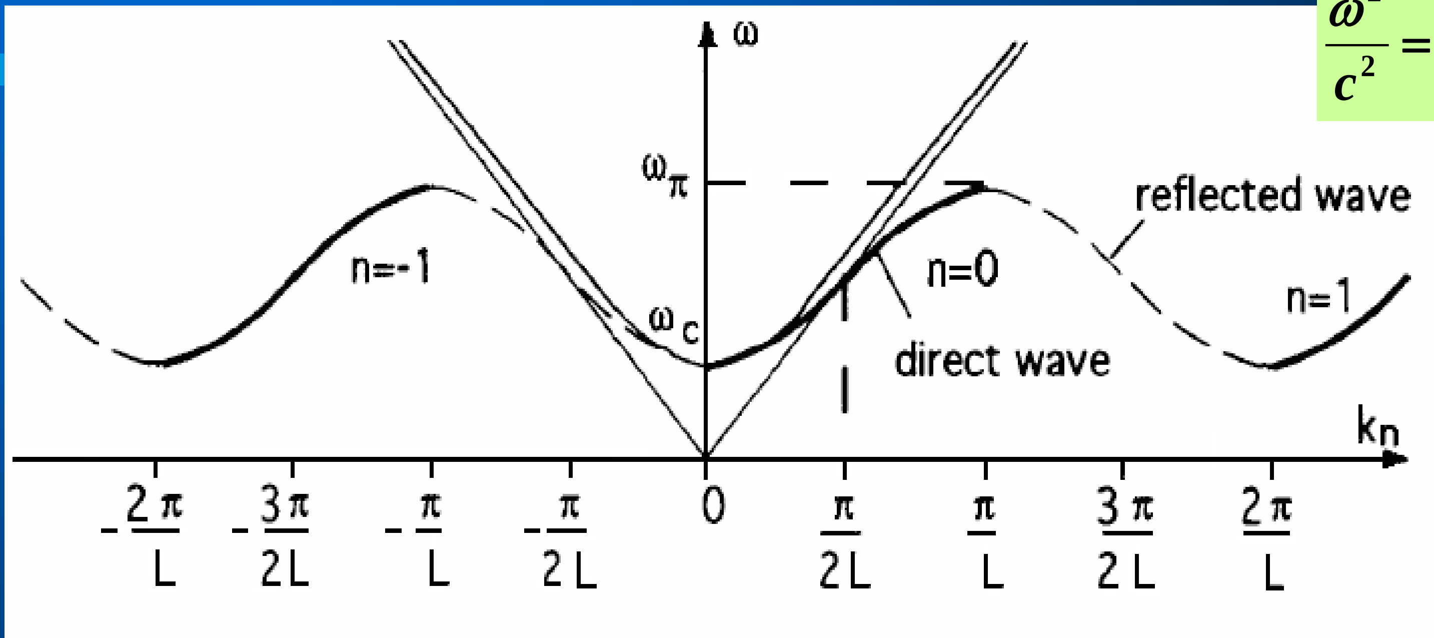
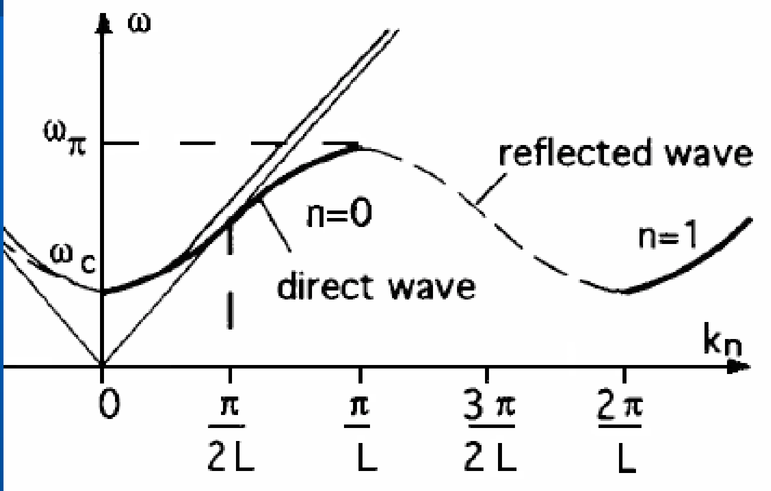


- Limited pass band of frequencies from ω_c to ω_π
- At both ends of the pass band v_g =0
- For a given ω, infinite series of space harmonics

  same group velocity, but different phase velocities

Travelling wave Accelerator

TWA operates with *n = 0*, wave propagates in only + *z* ($v_g > 0$)

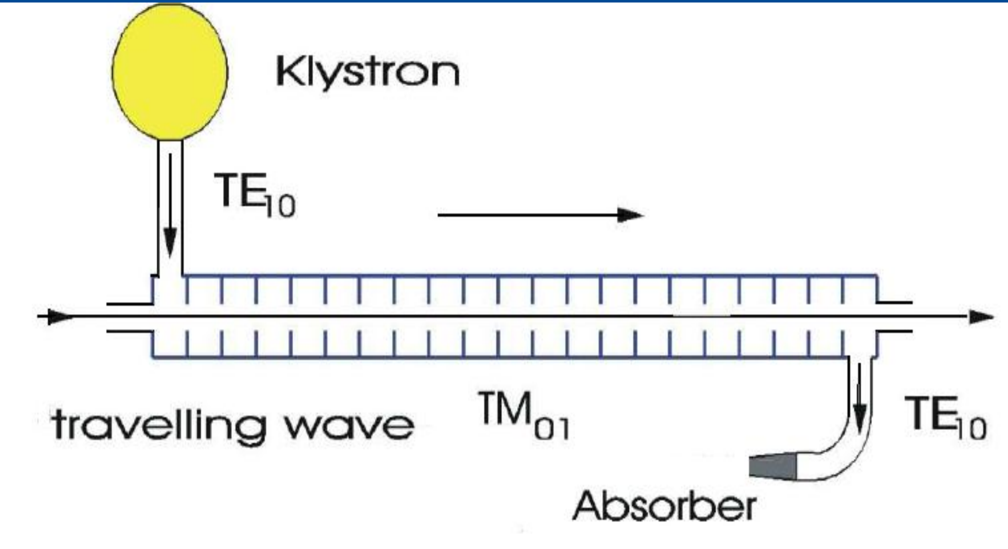


Used with short beam pulses particle velocity $v \sim c$, for electrons.

***kn* between $\pi/2L$ to $2\pi/3L$**

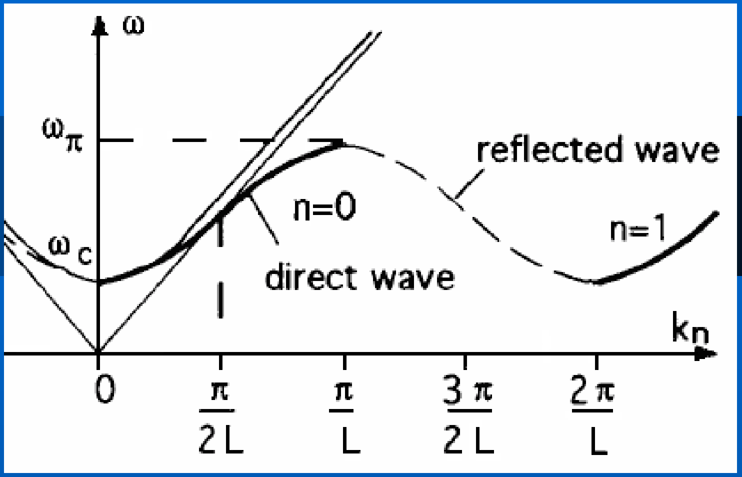
Generally operates in

- $\pi/2$ mode, L of each cell is equal to $\lambda_n/4$
- $2\pi/3$ mode L of each cell is equal to $\lambda_n/3$

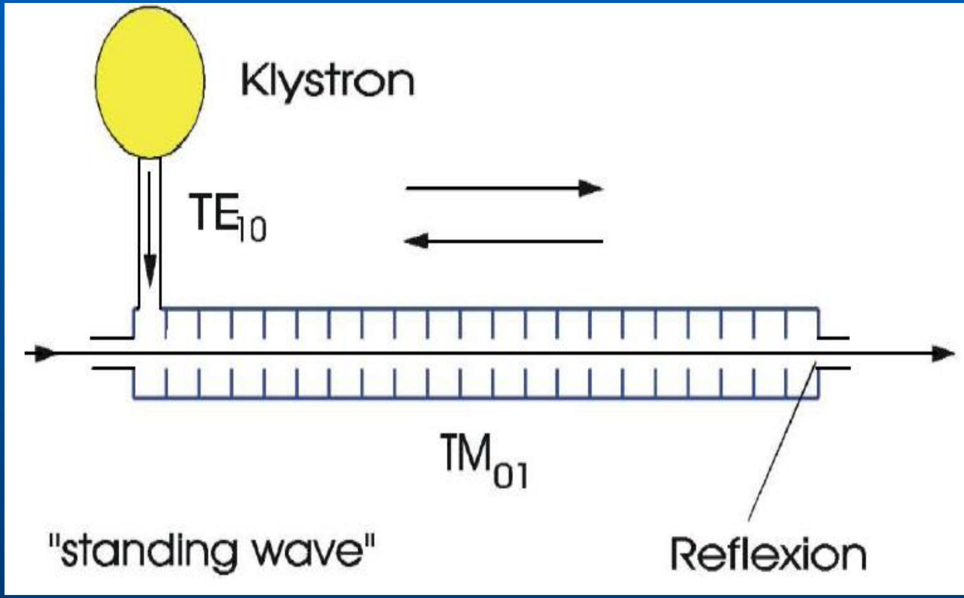


Standing wave Accelerator

Energy is reflected back and forth between the walls. Multiple reflections of the waves combine to create a standing wave pattern.



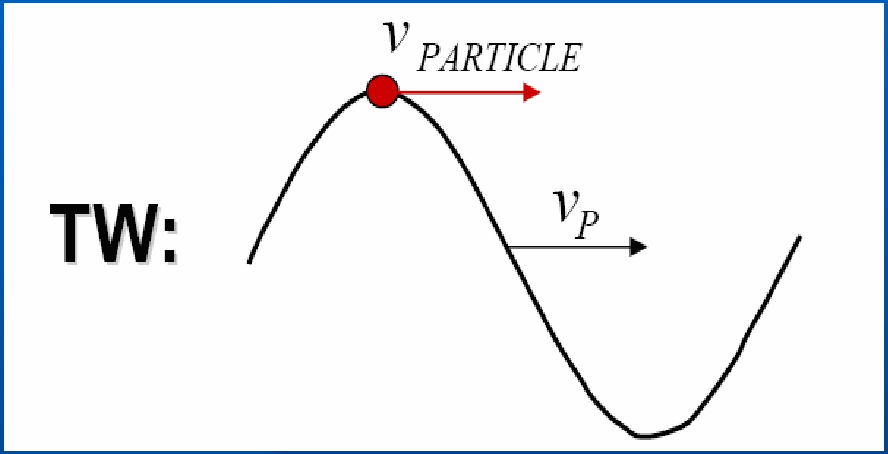
- Phase velocity *vp* is same where direct and reflected space harmonics join
- SWA operates at either ω_c lowest or at ω_π the highest frequency of the pass band.

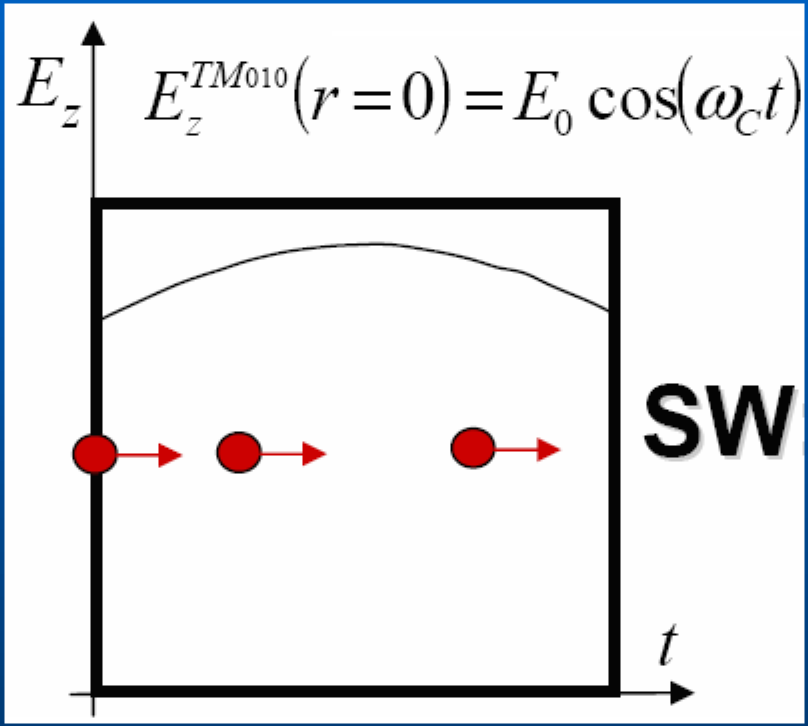


$$k_n = N\pi / L, \qquad N = 0, \ \pm 1$$

- In 0 mode fields in all the cells are in phase,
- In π mode fields in the adjacent cells are opposite in phase.

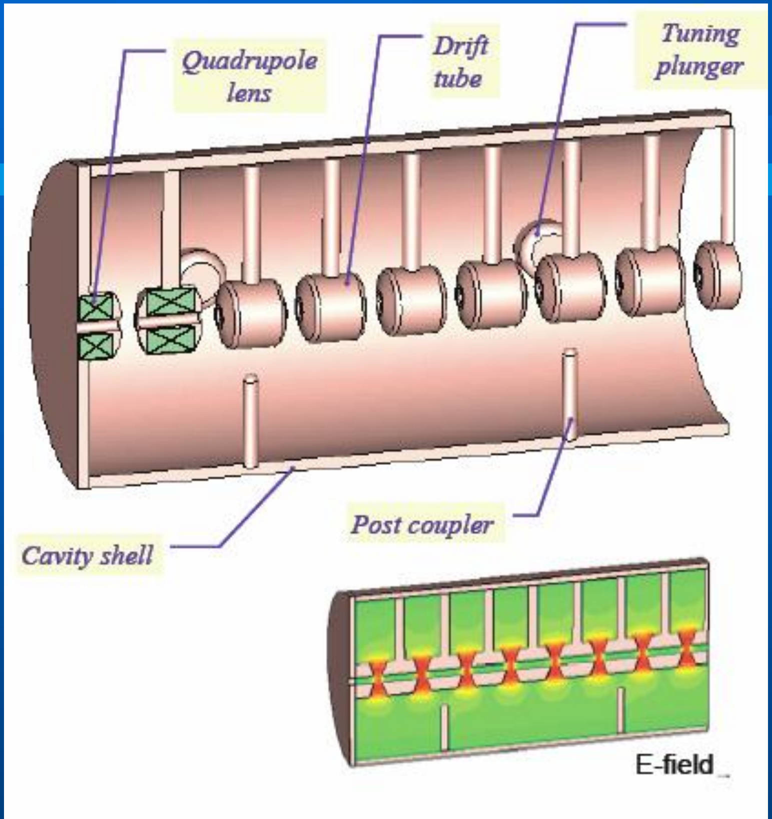
Travelling Wave and standing Wave

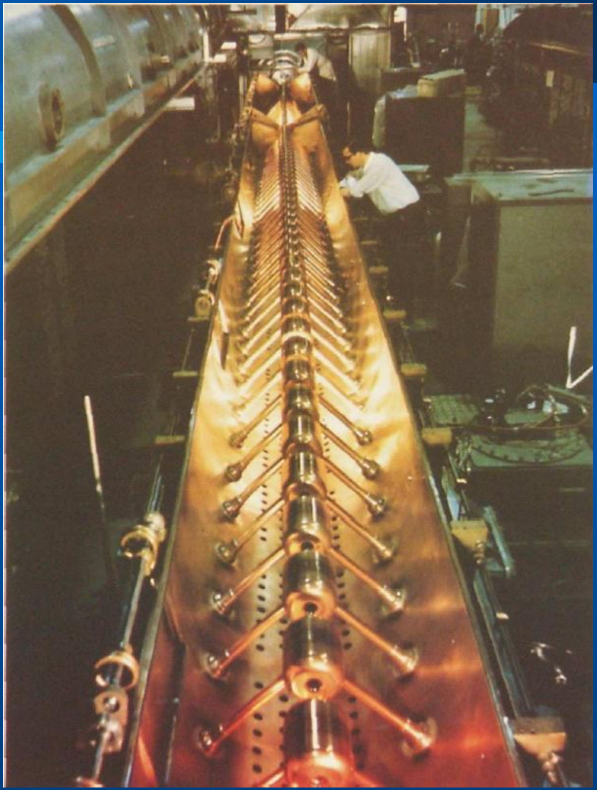




Standing wave Accelerator

Drift Tube Linac (DTL)





Focusing is provided by quadrupoles inside drift tubes.

Length of drift tubes increases with proton velocity.

Superconducting Cavity

$Q = 10^{10}$

$E = 20MV/m$

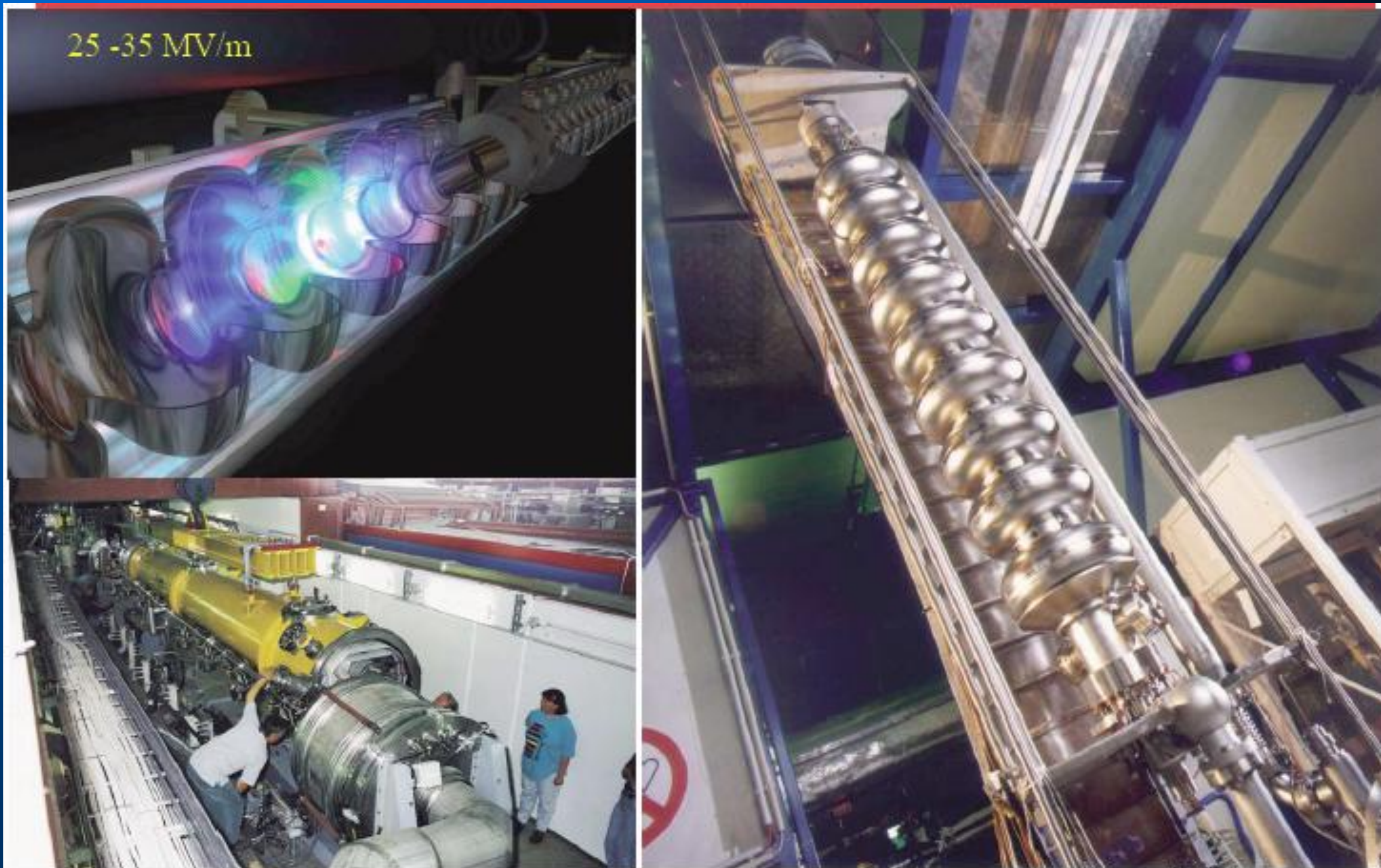
A bell with this Q would ring for a year.

- Very low wall losses.
- Therefore continuous operation is possible.
- Energy recovery becomes possible.

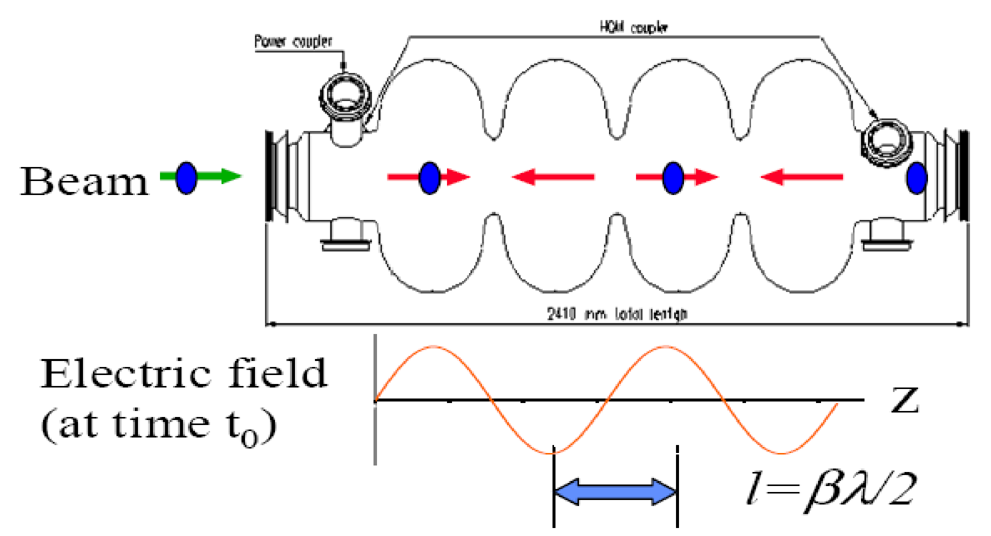
Normal conducting cavities

- Significant wall losses.
- Cannot operate continuously with appreciable fields.
- Energy recovery was therefore not possible.

Superconducting Cavity

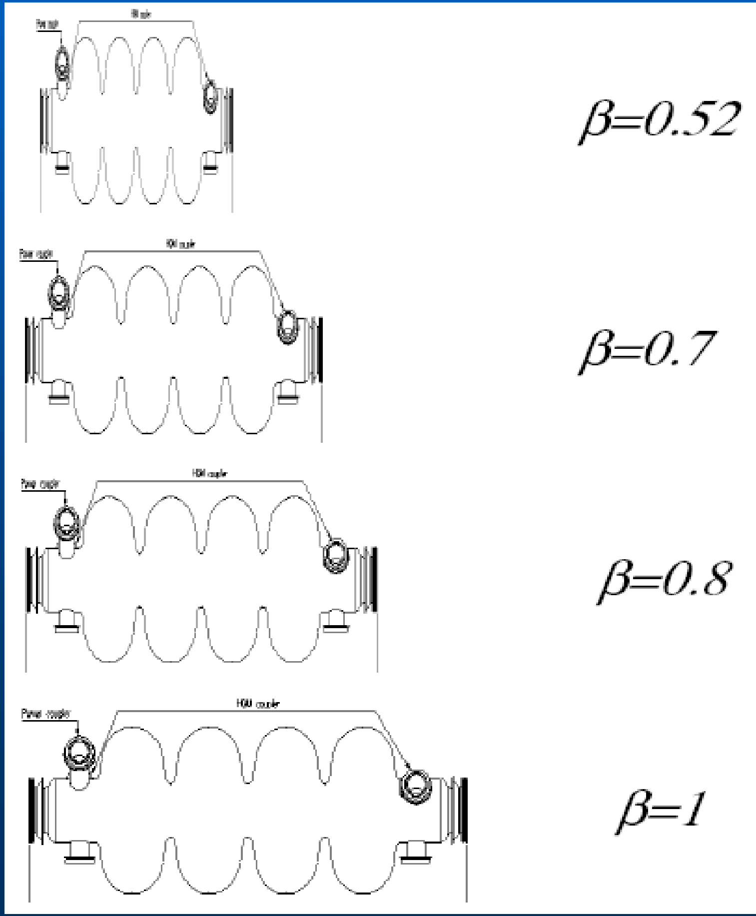


Electron vs Ions



Synchronism condition:

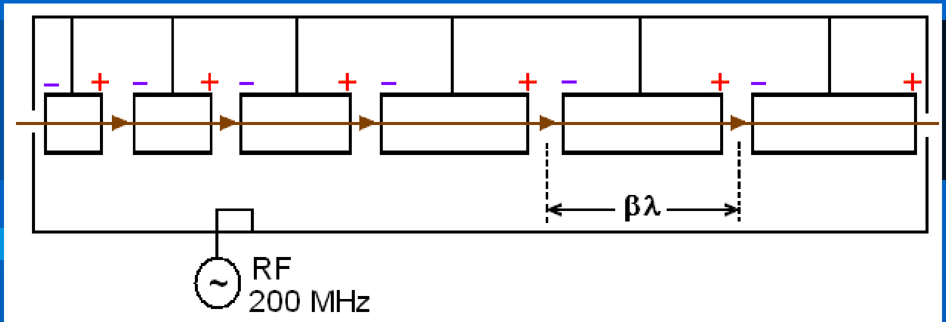
$$\frac{l}{\beta c} = \frac{1}{2f} \Rightarrow l = \frac{\beta c}{2f} = \frac{\beta \lambda}{2}$$



Electron linacs: $\beta = 1$, $l = \lambda/2 \rightarrow$ **an injector + a series cells of same length.**

Ion linac: **Sequence of different accelerating structures (changing cell length, frequency, etc) matched to the ion velocity.**

Principle of Phase stability

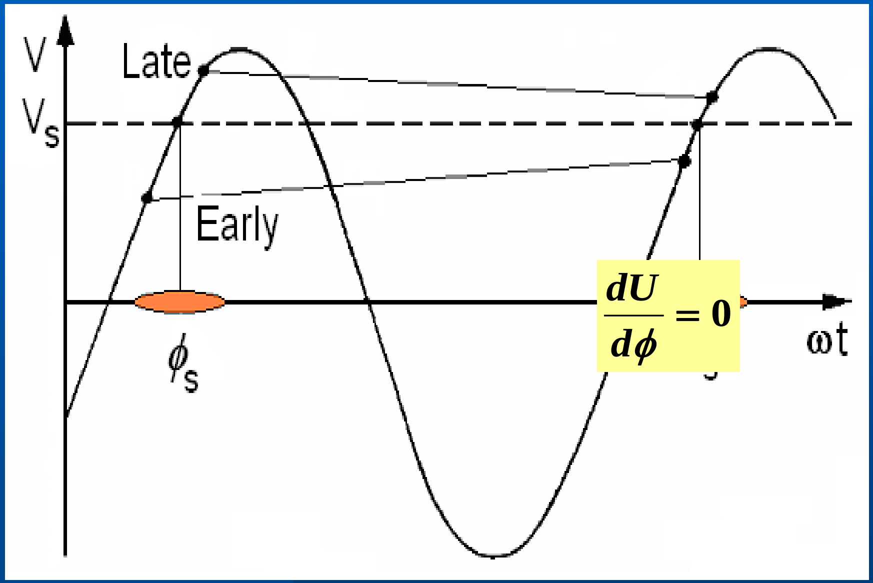


- In RF accelerators millions of particles in a bunch accelerated.
- The bunch has a finite width in phase & finite energy spread.
- A given geometry of the drift tube, energy gain must be exact.
- Can be satisfied for only one particle "synchronous particle".

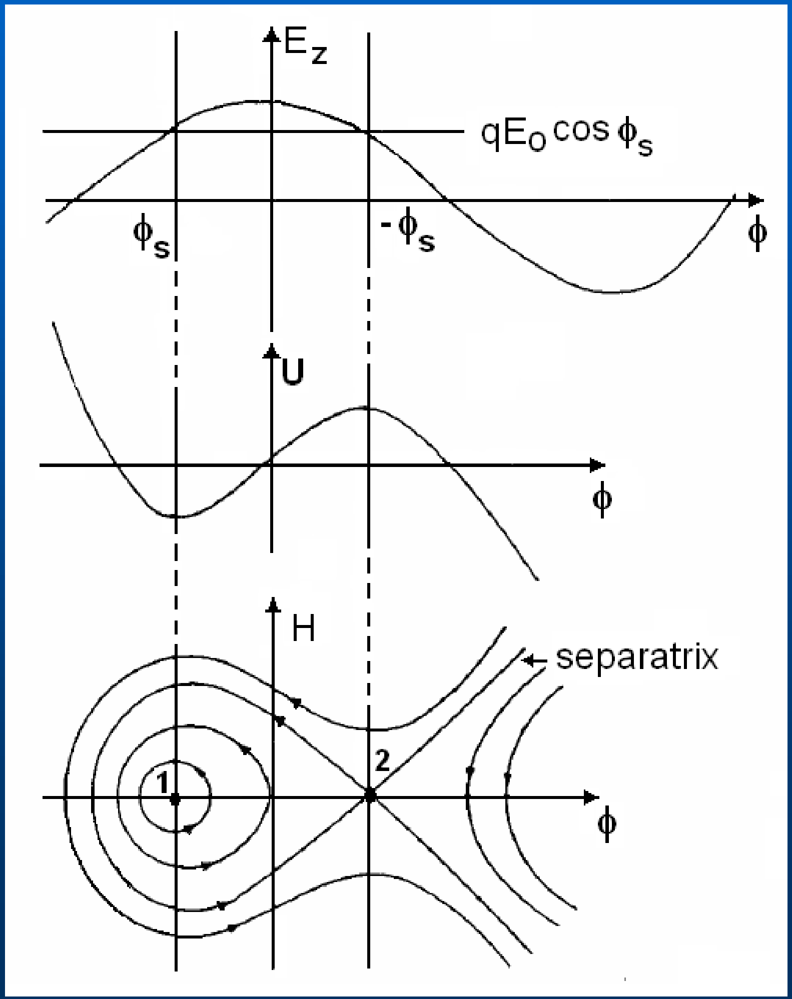
The principle of phase stability states that particles with phases and energies in the neighborhood of the synchronous particle are also accelerated by simply injecting them at the appropriate phase of the RF voltage.

Principle of Phase stability

$$H = -\frac{\omega}{2m_0c^3}\frac{(\Delta W)^2}{\beta_s^3\gamma_s^3} - qE_0T\left(\sin(\phi_s+\Delta\phi) - \Delta\phi\cos\phi_s\right)$$



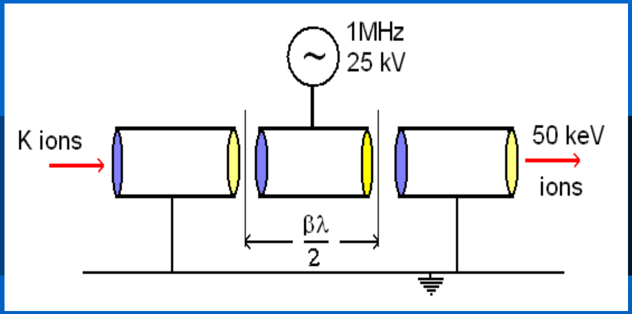
$$\frac{dU}{d\phi} = 0$$



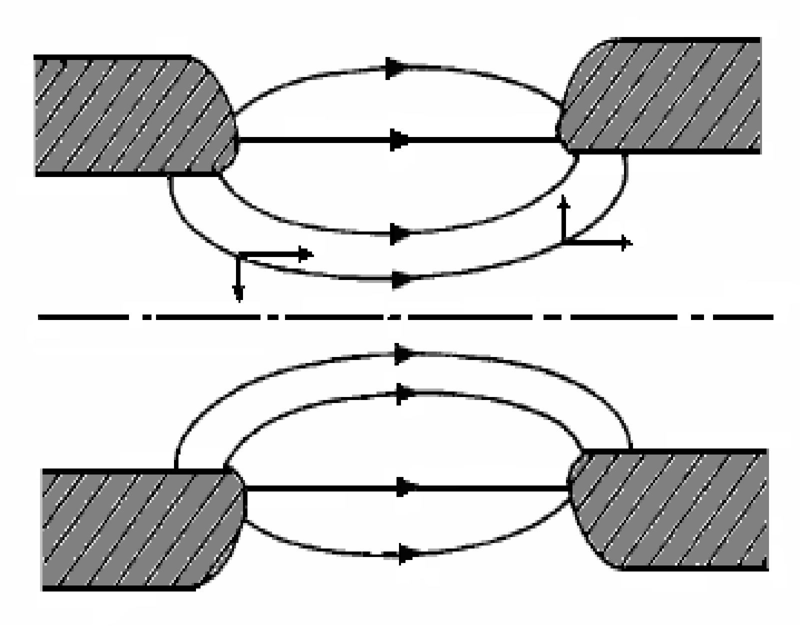
$$\Omega^2 = -\frac{\omega}{m_0c^3}\frac{qE_0T\sin\phi_s}{\beta_s^3\gamma_s^3}\Delta\phi$$

Synchrotron oscillation

Transverse Defocusing



Radial components of force
- focusing at the gap entrance
- defocusing at the exit



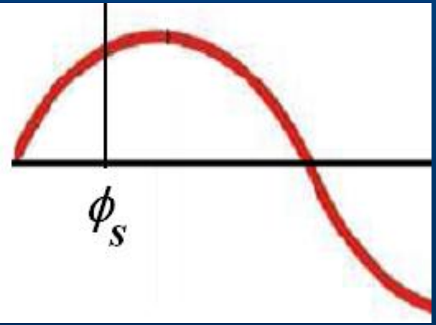
Particles have more energy at exit, spends less time, focusing dominates defocusing

Phase stability requires (ϕ_s < 0)

Particles experience
- less field in focusing region.
- more field in defocusing region.

$$r'' = -\frac{\omega q E_0 T \sin\phi_s}{2 m_0 c^3 \beta_s^3 \gamma_s^3} \cdot r$$

Overall defocusing unless ϕ_s +ve



Pause for cyclotron