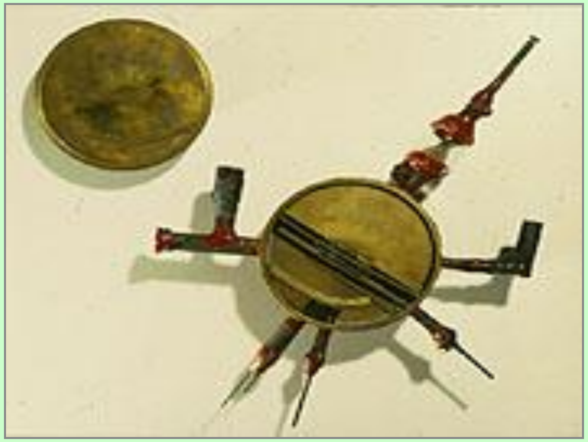
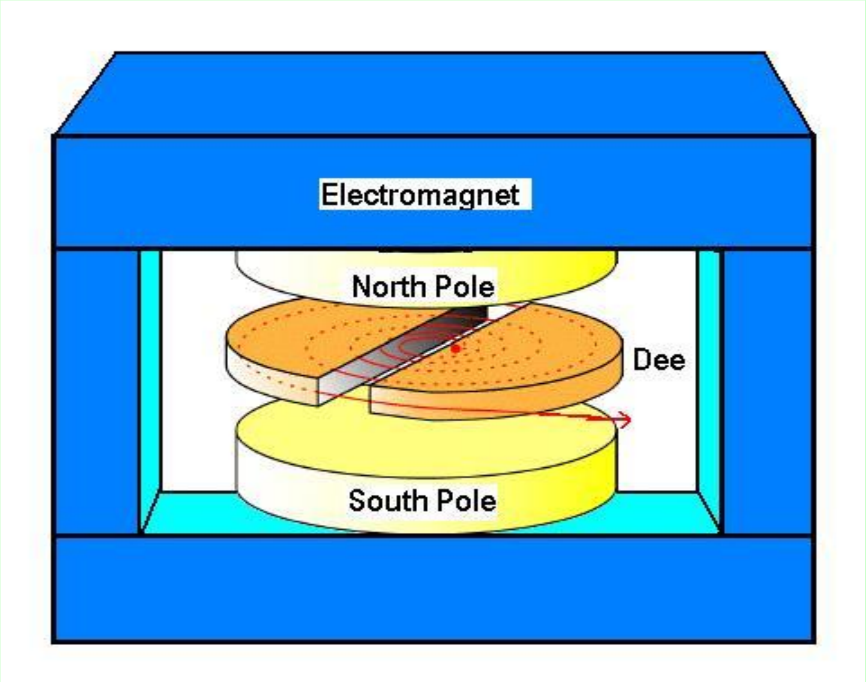
Cyclotron (Lawrence, 1929)

Based on the radio frequency acceleration and bending of charge particles in a magnetic field.

Same electrode is utilized again and again to accelerate particles.



E. O. Lawrence
1939 Nobel Prize

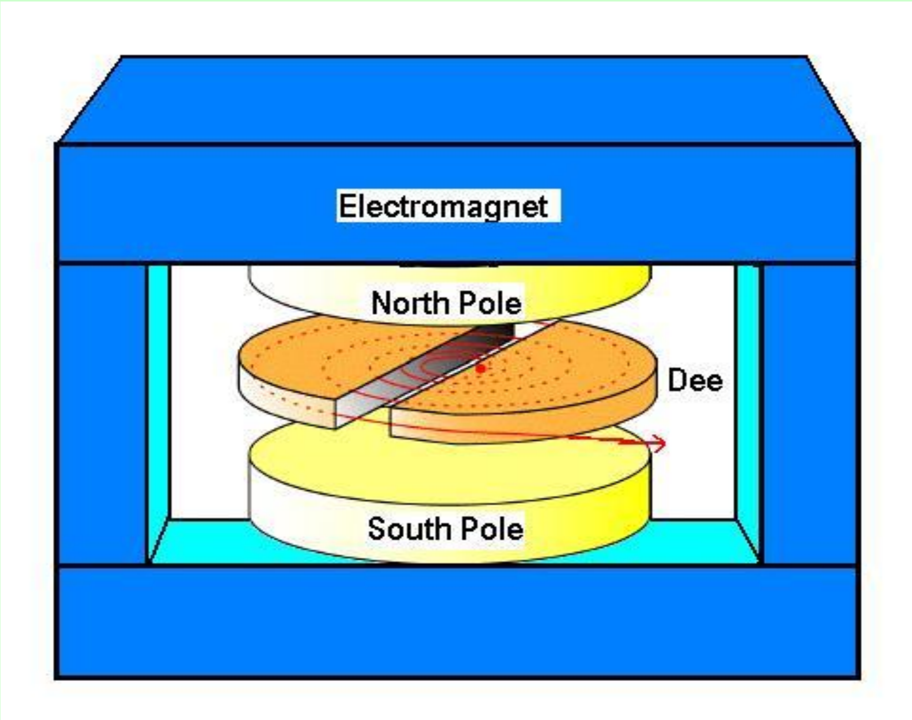


Basic equations

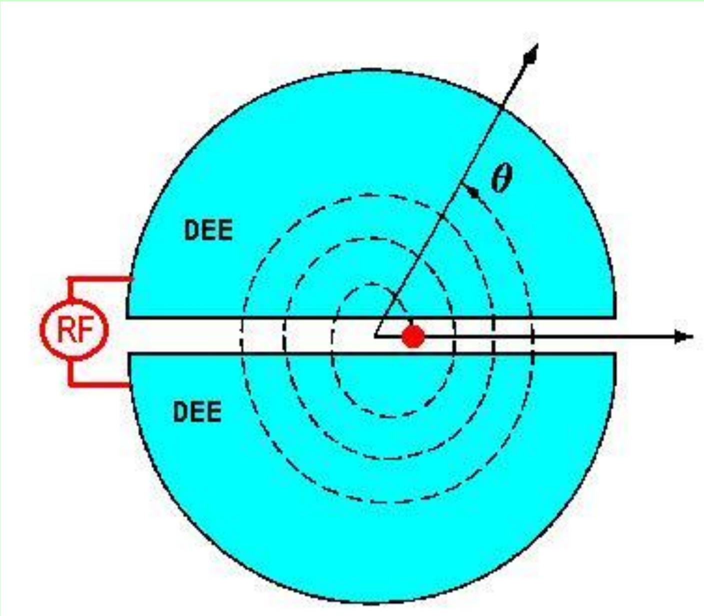
$$\frac{mv^2}{r} = qvB$$

$$p = qBr$$

$$\omega_p = \frac{qB}{m}$$



- Orbital frequency is independent of energy
- Orbit radius is proportional to momentum



Resonance Condition

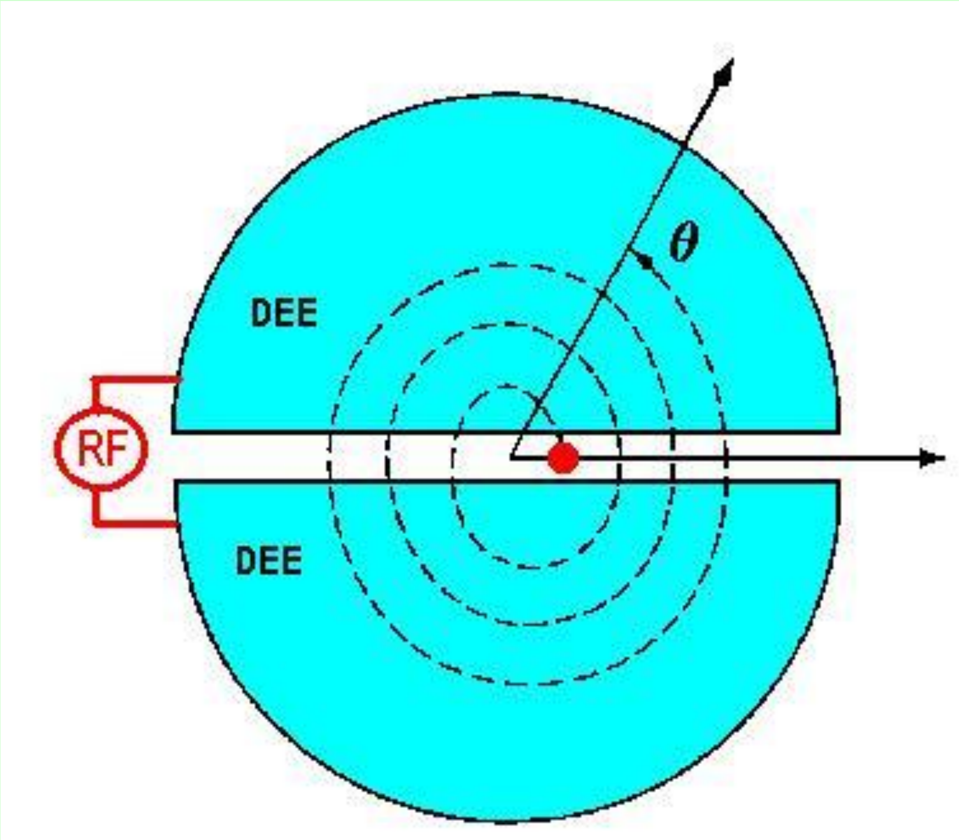
- Repeated acceleration needs resonance $\omega_{rf} = h\omega_p$
- For heavy ion harmonic number $h \geq 1$
- A discrepancy in resonance

  $\Rightarrow$ phase shift

$$d(\sin\phi) = 2\pi h n \frac{\omega_{rf} - \omega}{\omega}$$

- Limit of acceleration occurs at

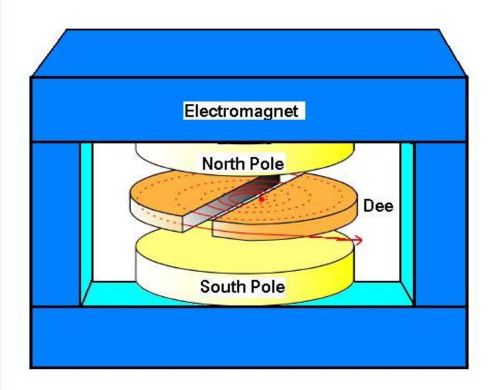
$$\phi = \pm 90^0$$



Few Relations

Kinetic Energy of particles

$$T = \frac{1}{2} m v_0^2 = \frac{q^2 B^2 R^2}{2m} = K \frac{Q^2}{A}$$

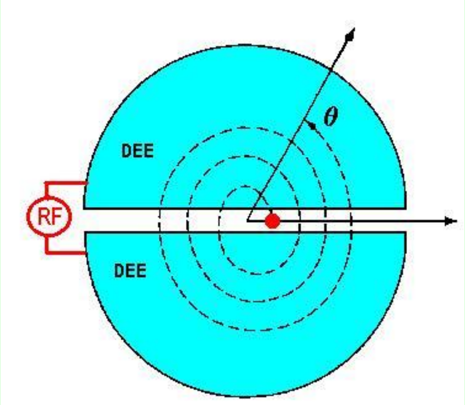


In terms of turn number

$$T = n \times 2 q V_D$$

K-value of a cyclotron

$$K = \frac{e^2 B^2 R^2}{2 m_p}$$



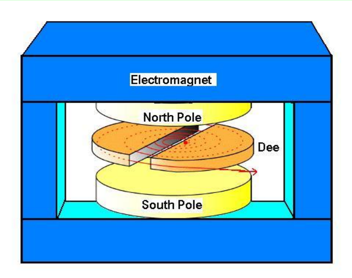
The path length traveled

$$L = \frac{4}{3} \pi n R$$

Requirement of Vacuum

$$N = N_0 \exp(-n \sigma L)$$

Classical Cyclotrons



In a uniform field particle orbits do not have axial stability.

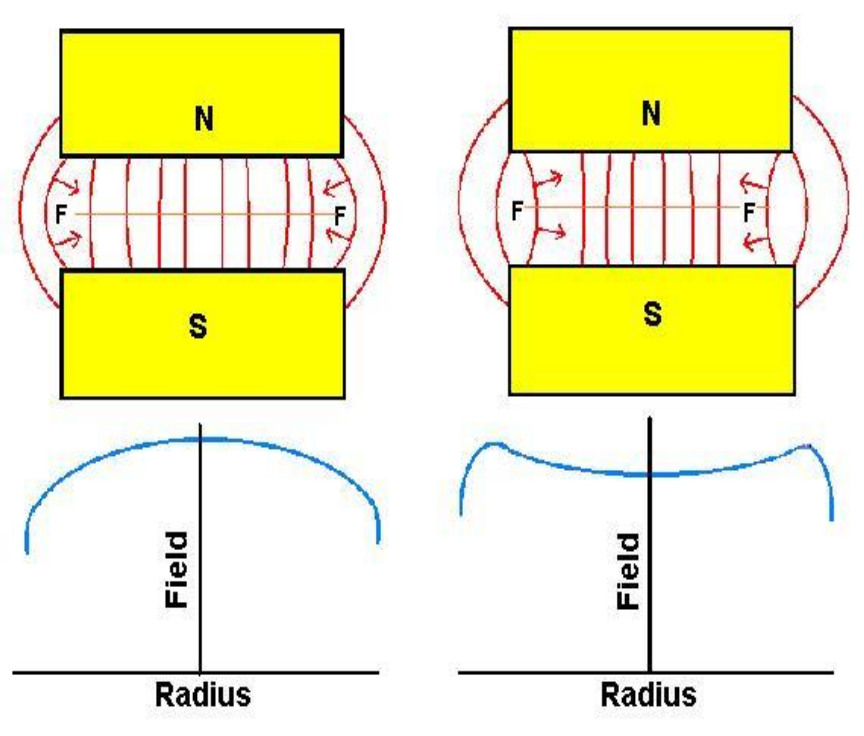
Average magnetic field with radius is decreased for vertical focusing

Off orbit particles execute SHM and remain confined

$$\ddot{z} + n\omega^2 z = 0$$

$$\ddot{x} + (1-n)\omega^2 x = 0$$

$$n = -\frac{r}{B}\frac{dB}{dr}$$



Betatron tunes determine orbit stability

$$\nu_z = \sqrt{n} \qquad \nu_r = \sqrt{1-n}$$

- Stability in both planes $\Rightarrow$ Real tune values

$$0 \le n \le 1$$

History

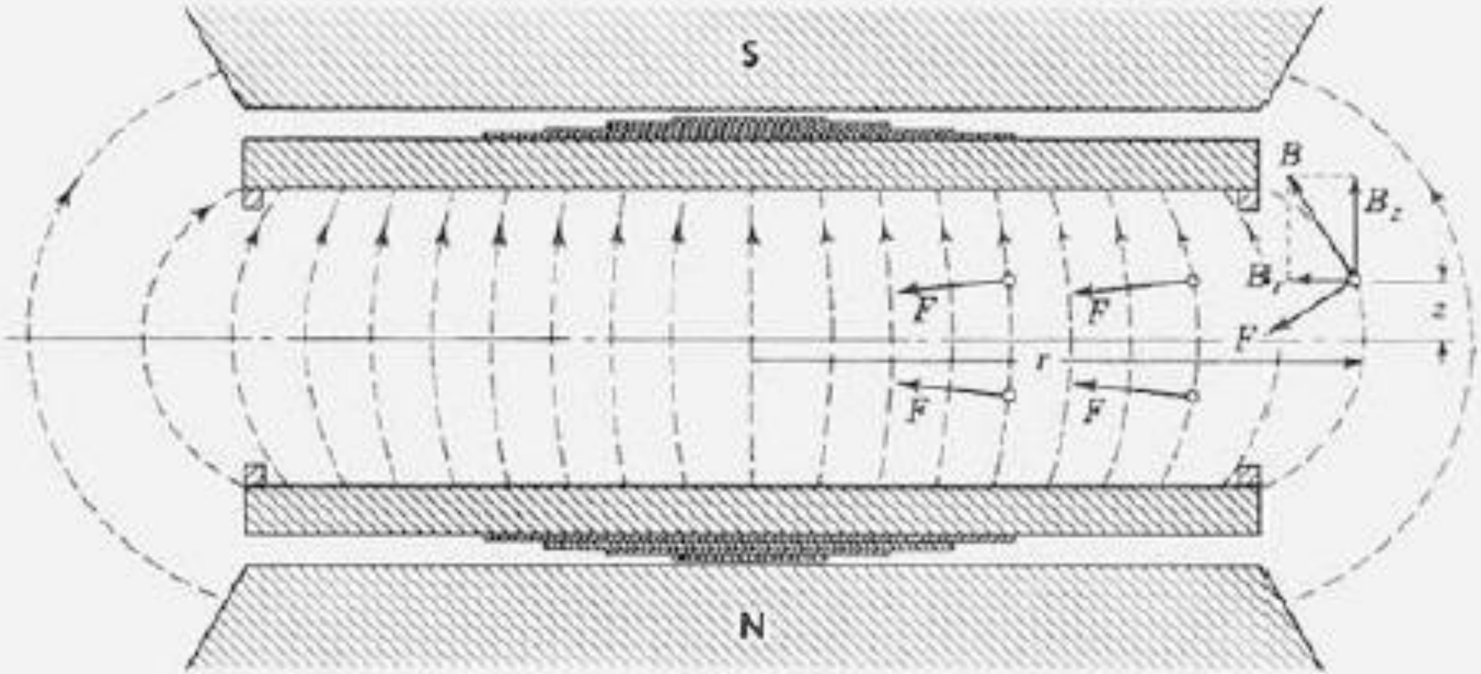


Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

Limitations of classical Cyclotrons

- Resonance $\Rightarrow B$ (fixed) as $r\uparrow$

$$\omega_{rf} = \omega_p = \frac{qB}{m}$$

- Axial focusing Requirement $\Rightarrow B\downarrow r\uparrow$

- **Relativity :**

As energy increases, mass increases

$$m = \frac{m_o}{\sqrt{1-\beta^2}} \Rightarrow B\uparrow \text{ as } r\uparrow$$

- The maximum K.E. is limited to

$$T \approx 4\sqrt{\frac{qV_D E_0}{\pi}} \approx 0.02\times E_0$$

History

H. E. Bethe and M. E. Rose predicted that Maximum T=12 MeV

Hans Bethe (1937) ***"it seems useless to build cyclotrons of larger proportions than the existing ones... an accelerating chamber of 37 cm radius will suffice to produce deuterons of 11 MeV energy which is the highest possible...***

Due to Bethe's influence, a paper appeared in 1938, seems to resolve the problem, was ignored for a decade. Paper was "*The Paths of Ions in the Cyclotron*" by L.H. Thomas

The paper was hard to understand. Ernest Courant (1952, who discovered the synchrotron and the principle of alternating gradient focusing) ***"A significant side benefit of inventing strong focusing was that it finally enabled me to understand what Thomas' paper was about."***

Solutions

Resonance $$\omega_{rf} = h\omega_p = h\frac{qB}{\gamma m_0}$$

1. Frequency Modulated Cyclotrons:

$$\gamma = 1 + \frac{T}{E_0}$$

outdated

$$\omega_{rf}\gamma = h\frac{qB}{m_0} = cont.$$

$$\omega_{rf}\downarrow r\uparrow$$

2. AVF cyclotrons:

Protons
Heavy ions

$$\frac{\omega_{rf} m_0}{qh} = \frac{B}{\gamma} = cont.$$

$$B\uparrow r\uparrow$$

3. Microtrons:

electrons only

$$\frac{\omega_{rf} m_0}{qB} = \frac{h}{\gamma} = cont.$$

$$h\uparrow r\uparrow$$

Azimuthally Varying Field Cyclotron

$$\frac{\omega_{rf} m_0}{qh} = \frac{B}{\gamma} = cont.$$

- Average field is increased with *r*

$$\langle B(r) \rangle = \gamma B_0 = (1 + \frac{T}{E_0}) B_0$$

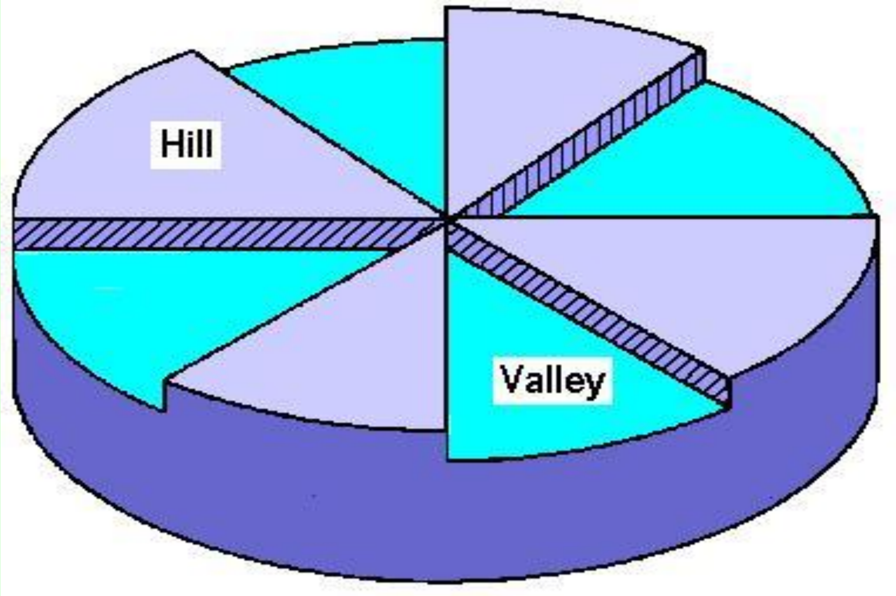
- Such a field gives axial defocusing

$$\Delta \nu_z^2 = -\frac{r}{\langle B \rangle} \frac{d\langle B \rangle}{dr} = -(\gamma^2 - 1)$$

Way to achieve axial stability

(L.H. Thomas 1938)

$$F = q\vec{v} \times \vec{B} = q \begin{bmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ v_r & v_\theta & v_z \\ B_r & B_\theta & B_z \end{bmatrix}$$



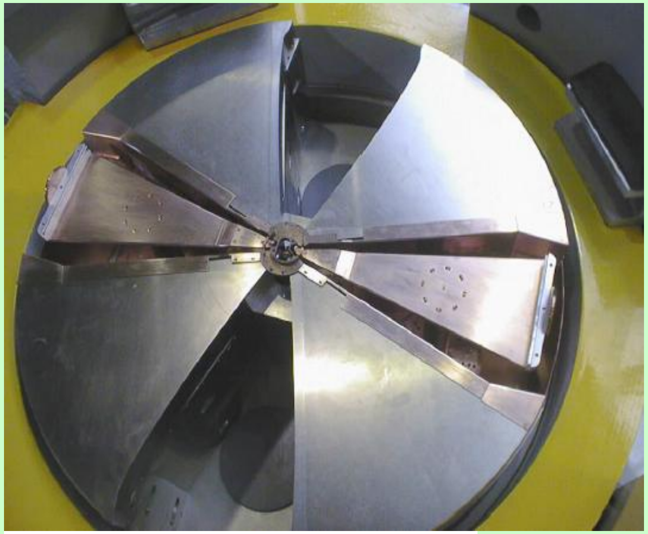
Thomas Focusing

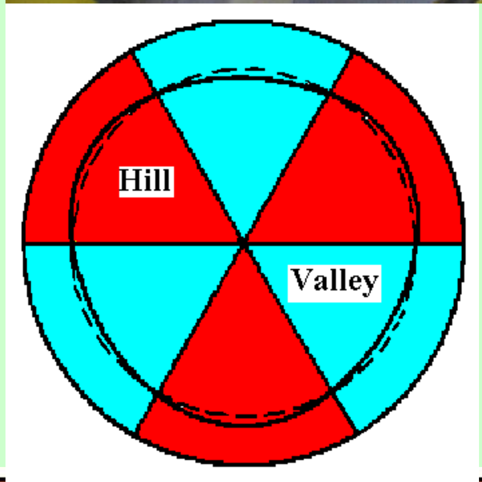
- Focusing force arises due to $v_r \times B_\theta$
- Strongest at the hill-valley boundary.
- Always focusing.
- Flutter

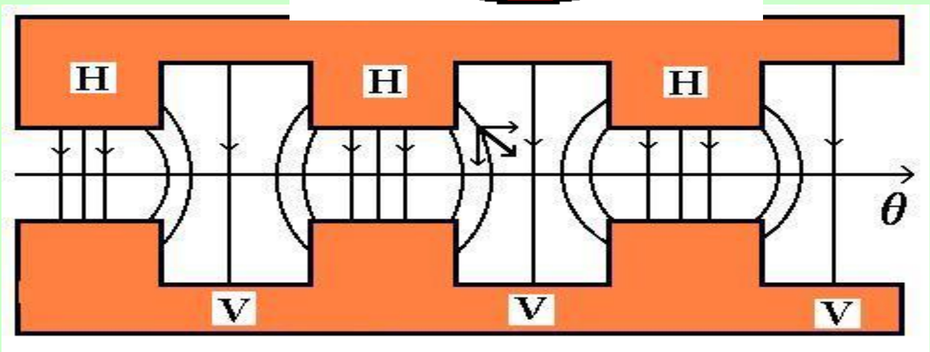
$$F^2 = \frac{\langle B^2 \rangle - \langle B \rangle^2}{\langle B \rangle^2}$$

- Thomas contribution

$$\Delta \nu_z^2 = F^2 = \frac{(B_H - \bar{B})(\bar{B} - B_V)}{\bar{B}^2}$$



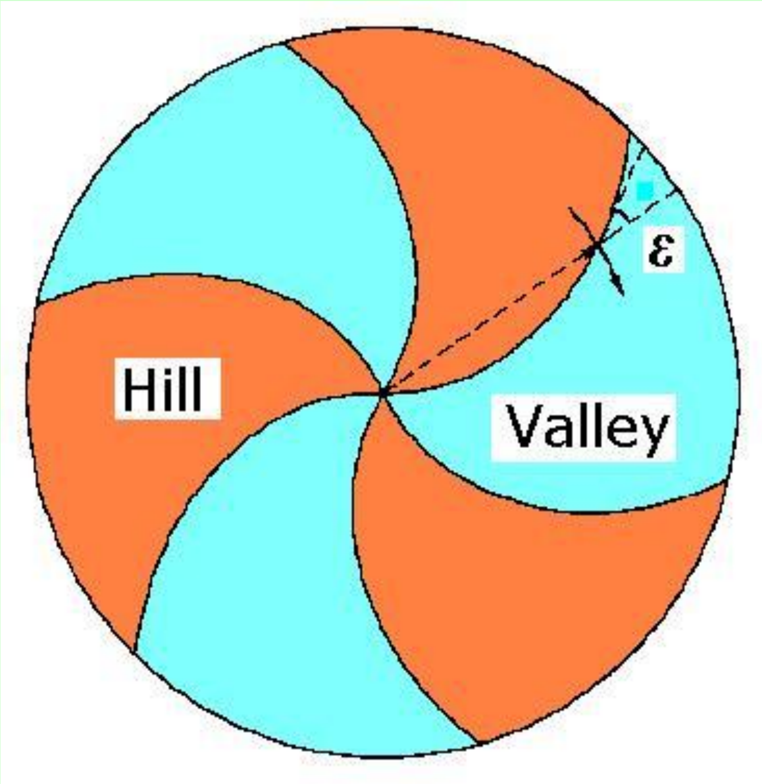




Spiral Focusing

Forces are focusing at one edge and defocusing at other edge.

$$v \times B_r$$



Net effect is focusing due to AG principle.

$$\Delta \nu_z^2 = 2F^2 \tan^2 \varepsilon$$

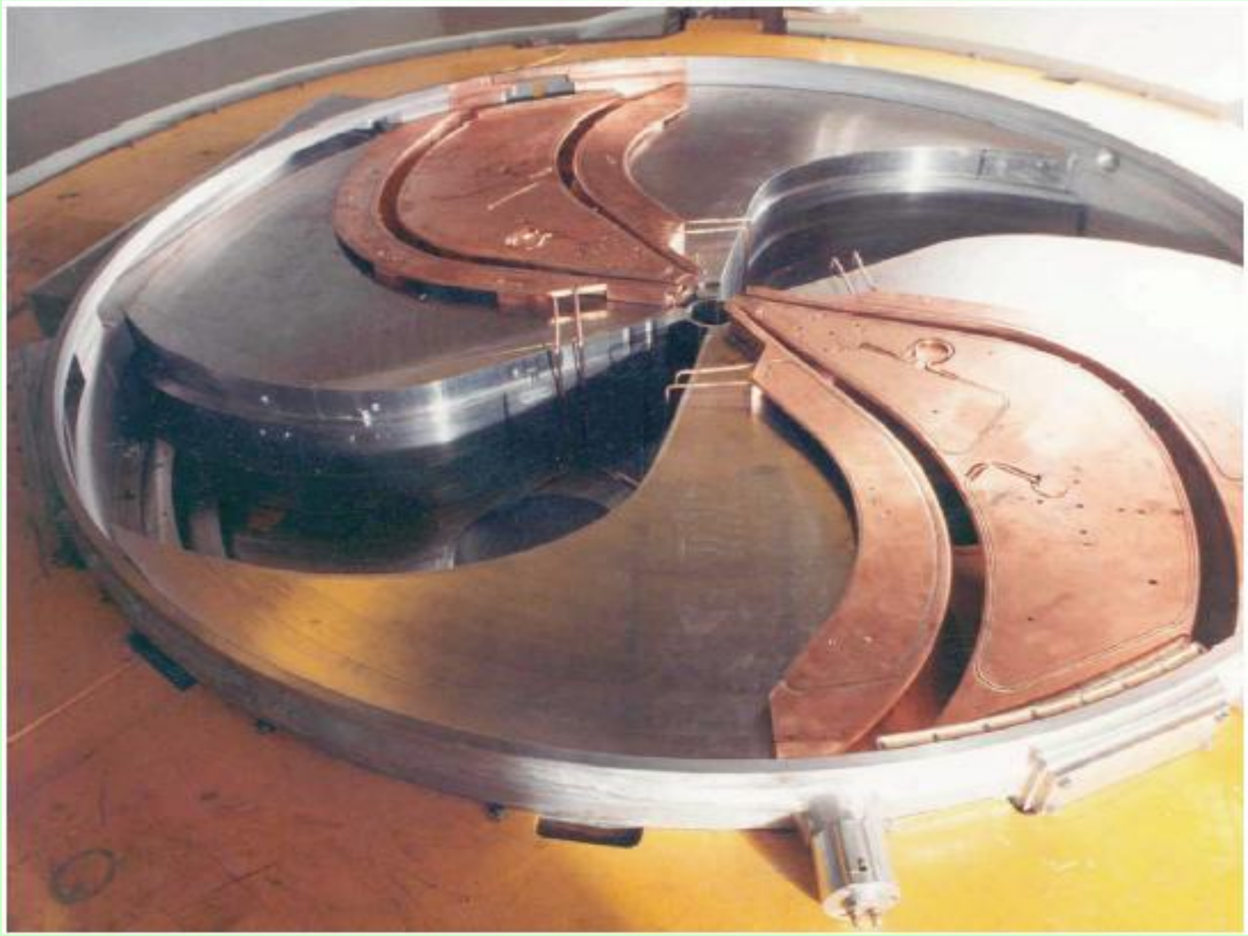
Adding all the contributions

$$\nu_z^2 = -(\gamma^2 - 1) + F^2(1 + 2\tan^2 \varepsilon)$$

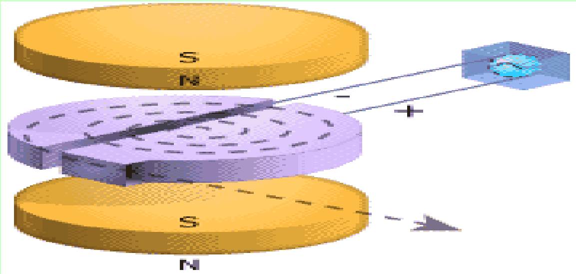
$$\nu_r^2 = \gamma^2 + ...$$

For focusing |second term| > | first term|

AVF Cyclotron



Extraction From Cyclotron



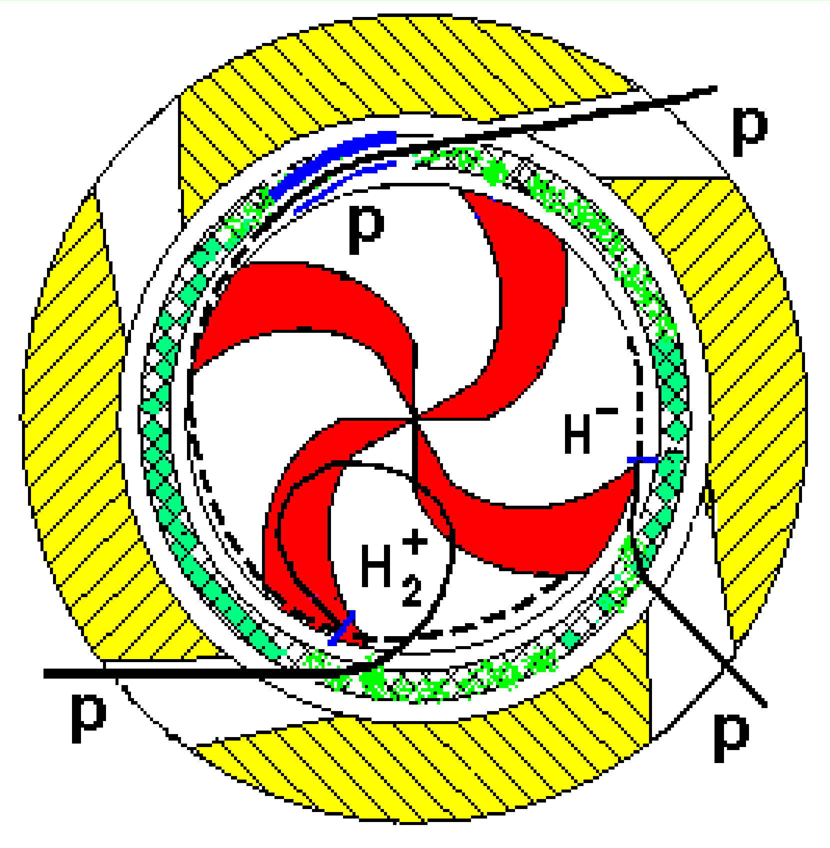
1. Electrostatic Channel

$$qvB - F_{out} = \frac{mv^2}{R}$$

3. Extraction by stripping for

H^- H_2^+

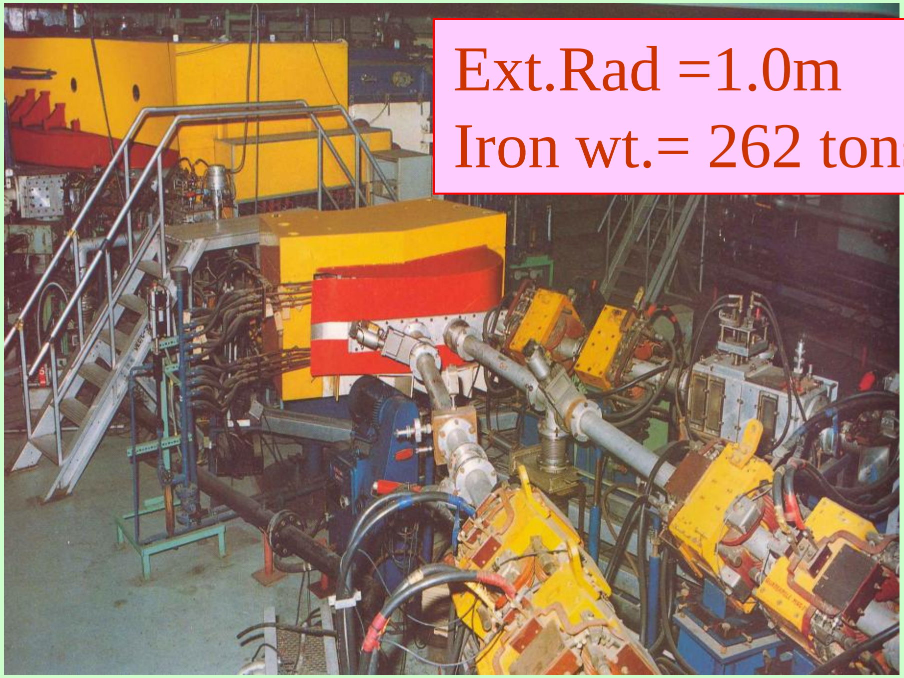
$$\frac{mv^2}{R} = qvB$$



•A thin foil (carbon) is inserted at a suitable radius.

•Stripping changes the radius of curvature.

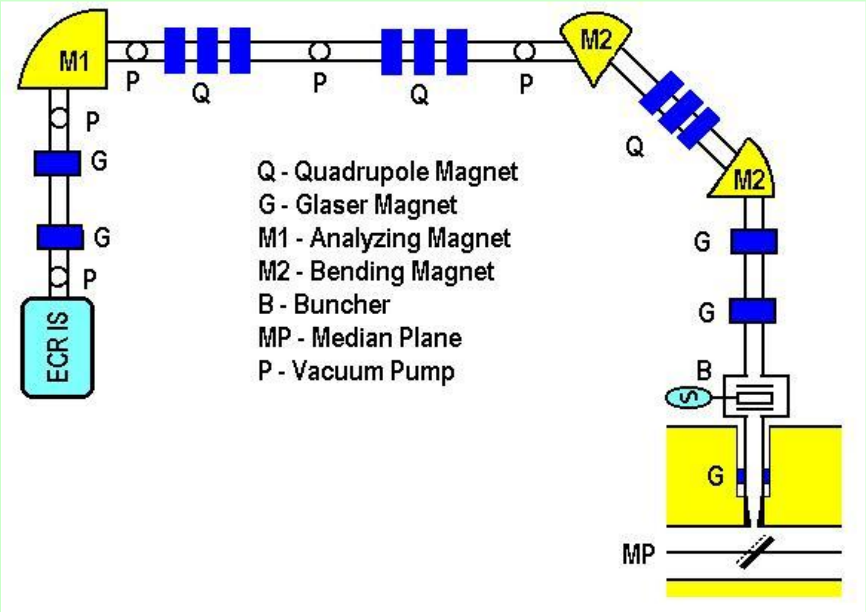
Example # 1 VECC at Kolkata



Ext.Rad =1.0m
Iron wt.= 262 tons

K=130 MeV
No. of sectors =3 spiral
No of Dee = 1

protons: 6-60 MeV
deuterons: 12-65 MeV
heavy ions: 130 Q^2/A MeV.



VEC provides:
- light ions internal PIG ion source
- heavy ions with ECR ion source

Research in: Nuclear Physics, Condensed Matter Physics, Material Sciences

Example # 2 Superconducting Cyclotron

Magnetizing force is supplied by sc-coils, consuming little power.

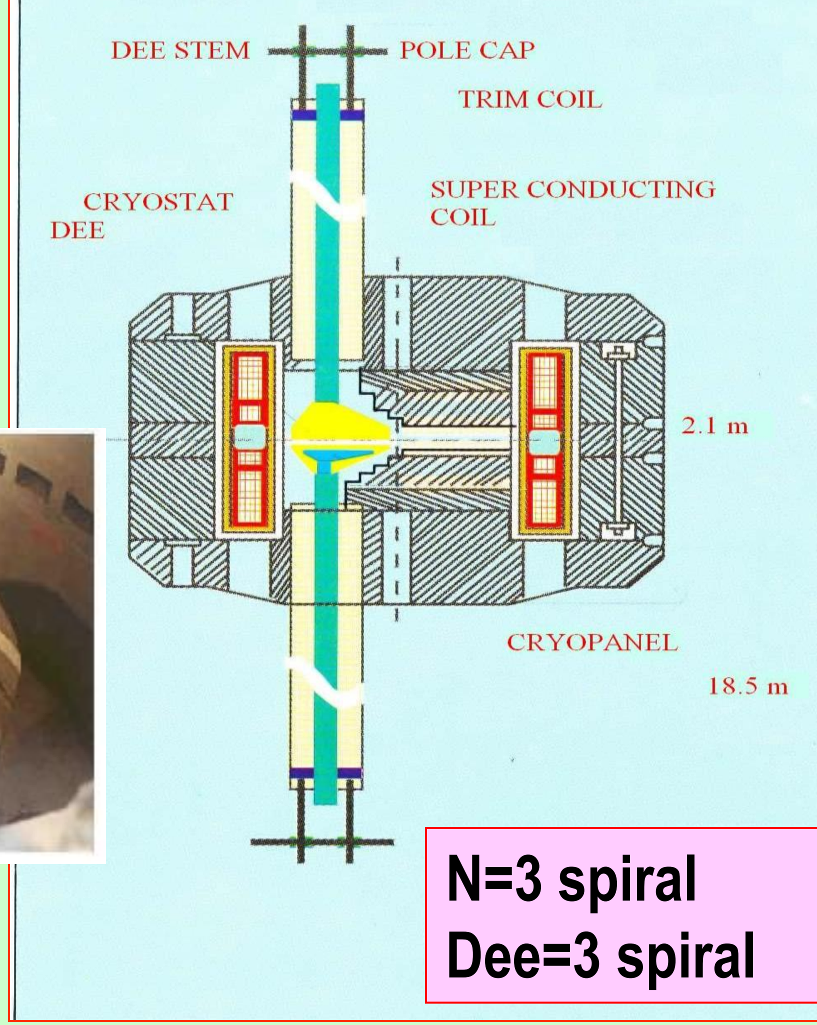
SC coils ~NbTi
High B ~ 5-6 Tesla

$$T = \frac{q^2 B^2 R^2}{2m}$$

$$B = \mu_0 (H + M)$$
$$H \,\alpha\, NI$$

K-500 at MSU, USA
K-500 at VECC, Kolkata

Ext. Rad =0.67m
Iron wt. = 100 tons

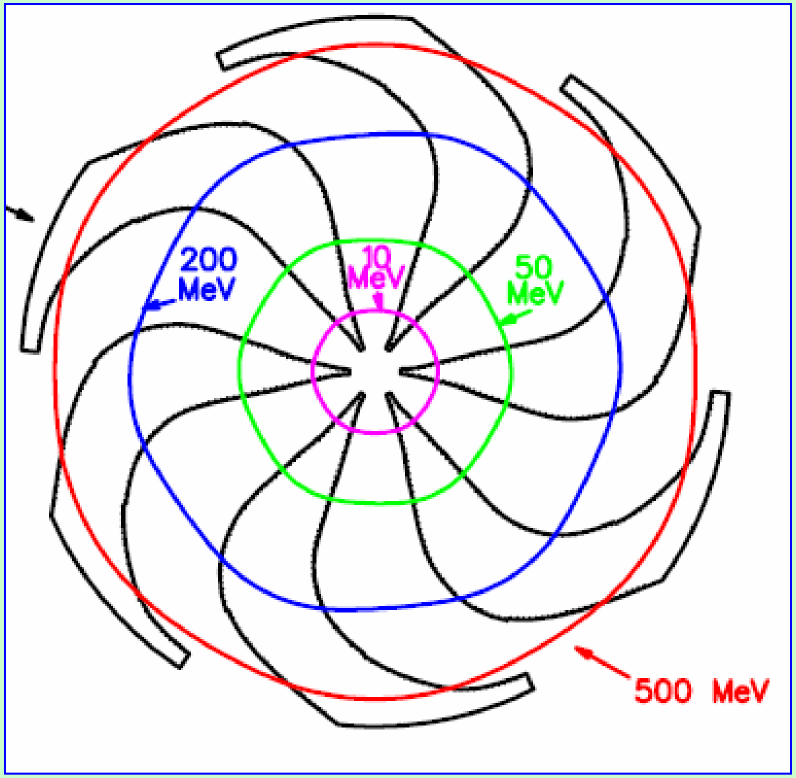


N=3 spiral
Dee=3 spiral

Example # 3 Largest AVF Cyclotron

- TRIUMF: protons 520 MeV for pion production
- N=6 spiral No. of dee = 2, 180 deg
- H^- ions are injected from external ion source.
- Extraction: by stripping (Carbon foil)

Diameter =12m
Iron wt. = 4000 tons



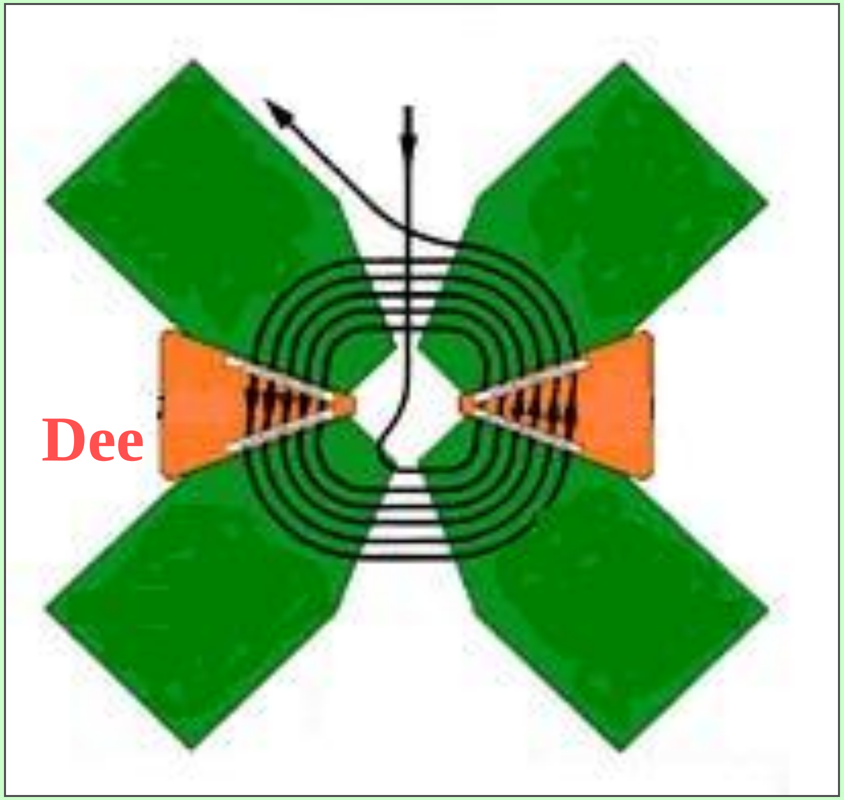
Example # 4 Separated Sector Cyclotron

SSC is also an AVF isochronous cyclotron where magnet sectors are separated by empty valleys. Used for high current.

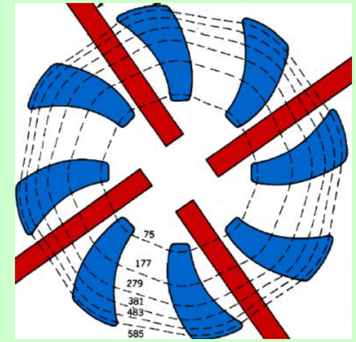
$$\Delta \nu_z^2 = F^2 = \frac{(B_H - \overline{B})(\overline{B} - B_V)}{\overline{B}^2}$$

Focusing force will be max when $B_V = 0$. Used for high current.

Powerful accelerating structures are put between the sectors

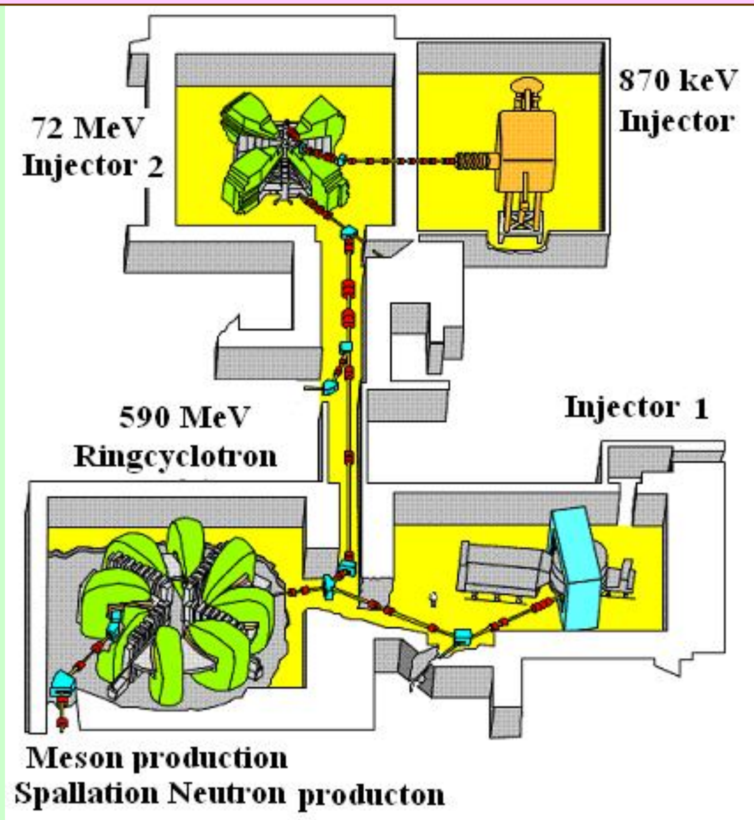


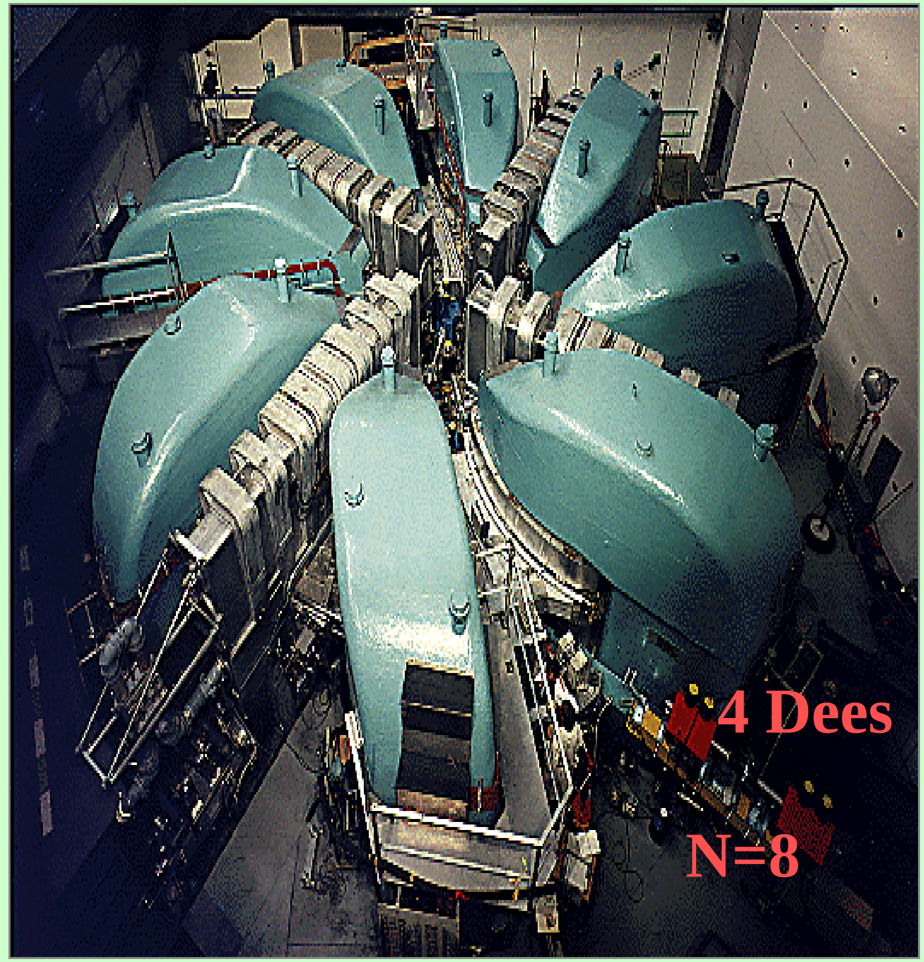
Example: Separated Sector Ring Cyclotron



PSI Machine: proton ~ 590 MeV, I ~ 2mA

Used for the production of pions and spallation neutrons.





Example # 5 Medical Cyclotrons

10-30 MeV p, d
I=100-500 μA

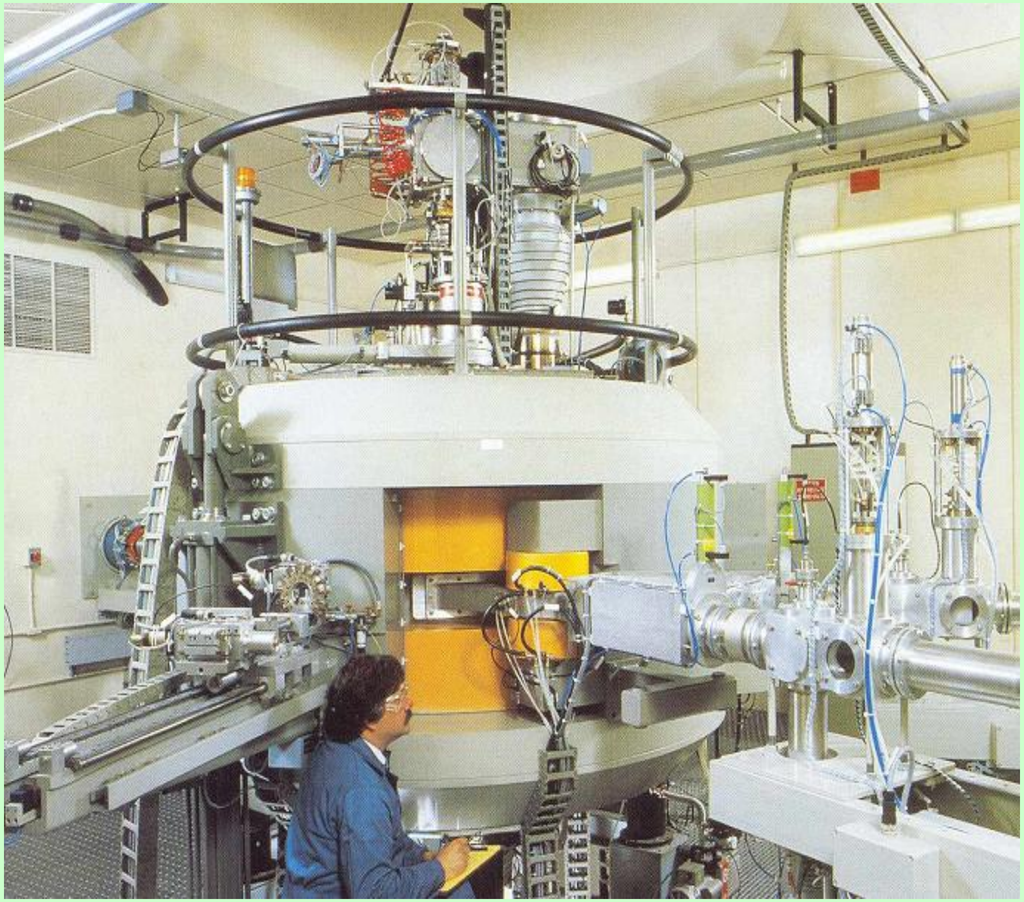
Production of radioisotopes :

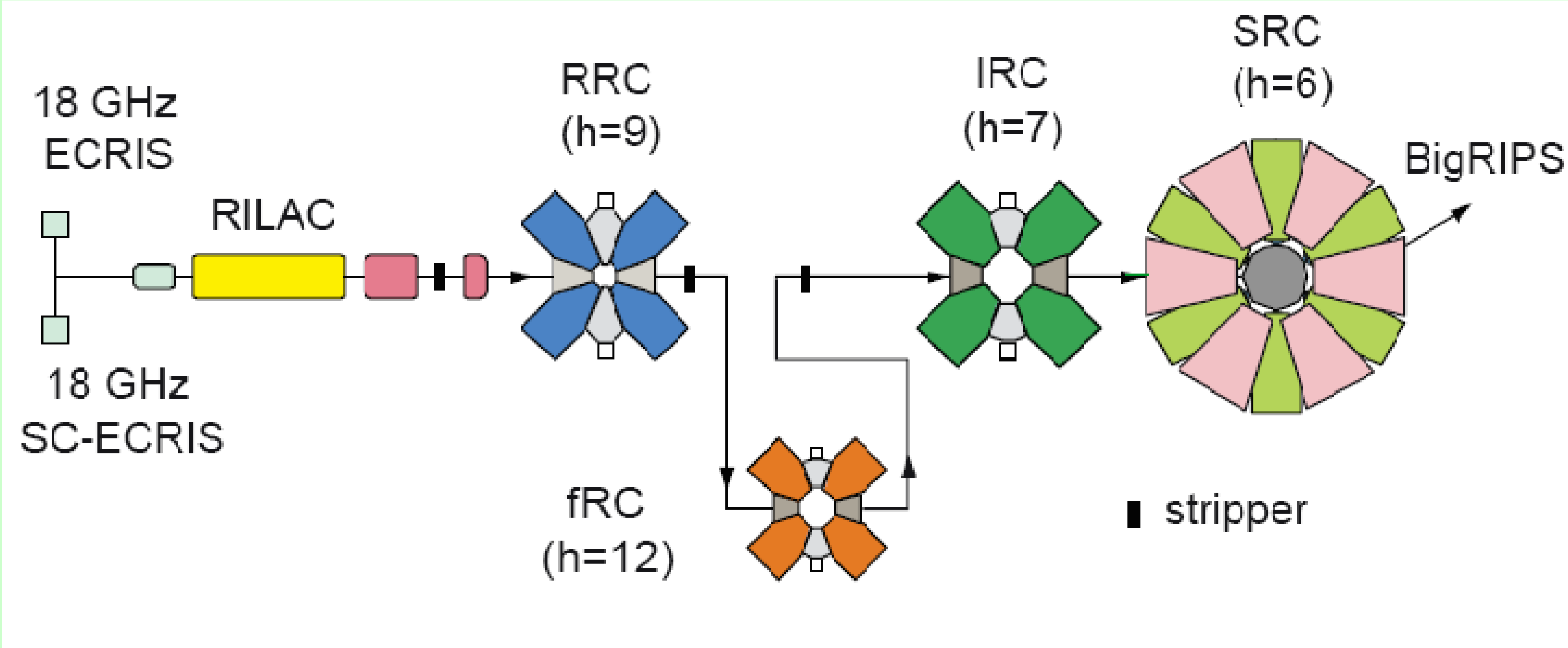
Thallium (Tl-201),

Gallium (Ga-67),

Indium (In-111)

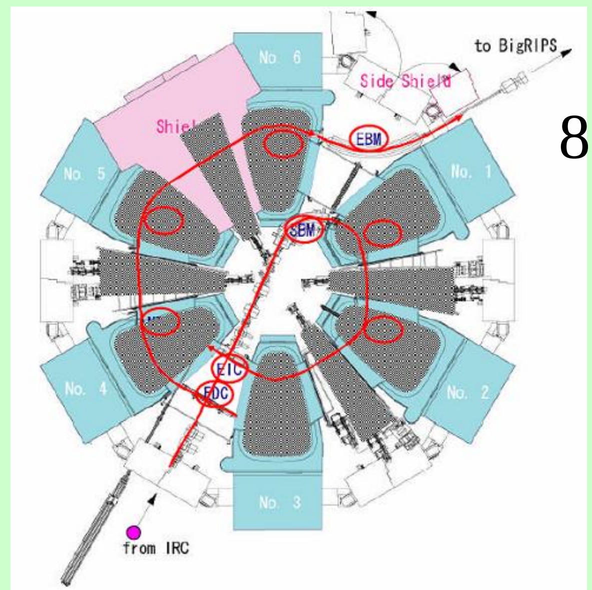
Production of PET isotopes





K78 MeV AVF
RRC- K540 MeV,
fRC- K570 MeV,
IRC-K980 MeV
SRC K2,600 MeV.

$^{238}U^{86+}$ ions 345 MeV/nucleon



8300 tons

Thanks