Alternate Phase Focusing in sequence of independent phased resonators as SC Linac Boosters for proposed ANURIB Facility

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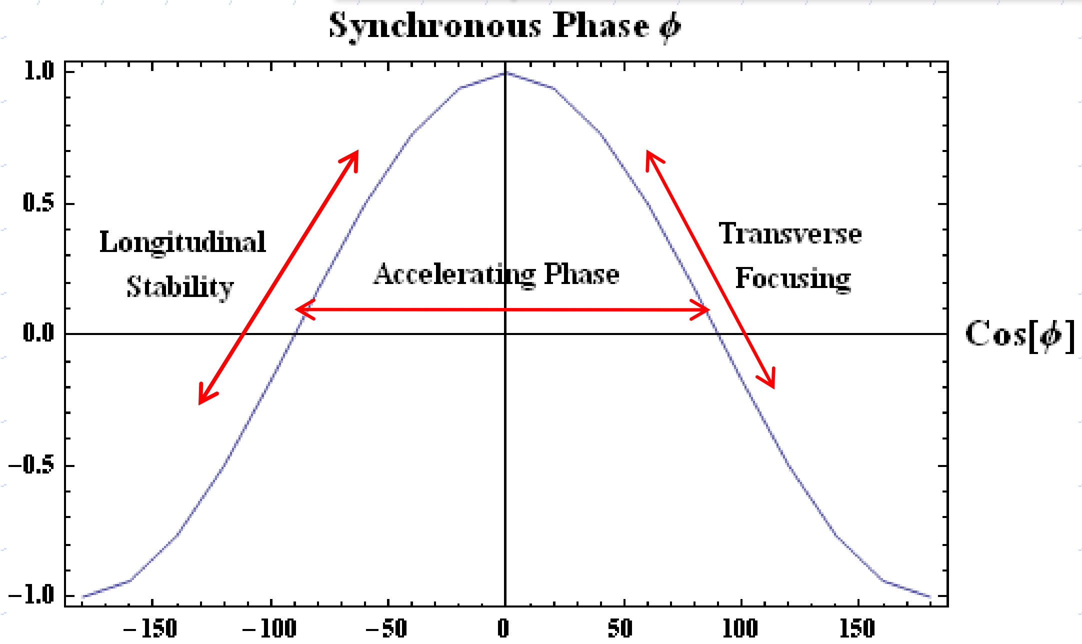
Concept of Alternate Phase Focusing



Synchronous phase of linear accelerator chosen in the rising part of RF voltage waveform to ensure longitudinal bunching which in turn induces transverse defocusing.

Need for external transverse focusing elements such as quadrupoles or solenoids.

Alternate phase focusing **(APF)** is based on periodic changes of RF field synchronous phase sign to maintain longitudinal and transverse beam stability simultaneously for a series of accelerating gaps. First discovered by *Good[Phy. Rev. 92(1953)]* and *Fayenberg [Zh. Tekh. Fiz. 29 (1959)].* Main types include

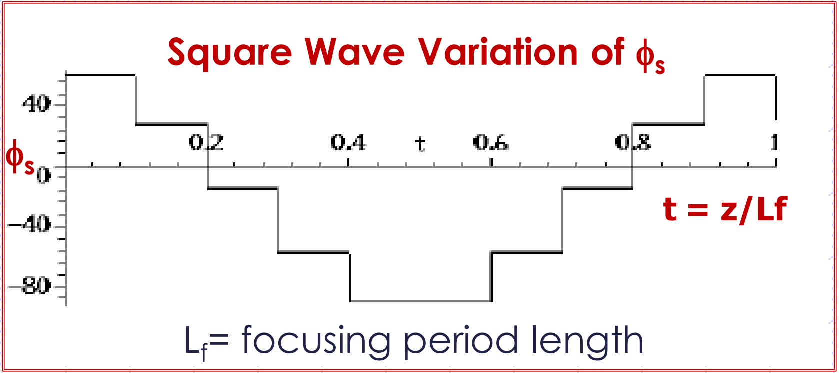
(a) Symmetric APF: N=2 (ϕ_1, $-\phi_1$)
(b) Asymmetric APF: N=2 (ϕ_1, $-\phi_2$)
(c) Modified APF: N> 2 and like in AAPF phases are not equal
where, N denotes number of accelerating gaps per focusing period.

SAPF suffered from too small longitudinal acceptance.

Revisiting APF structure started after work by V. V. Khushin in 70's on Asymetrical Alternate Phase Focusing (AAPF) which have larger longitudinal acceptance.

APF have been realized in multi-acceleration gaps essentially in long drift tube structures (*Design of APhF-IH Linac for a Compact Medical Accelerator, V Kapin et,al, NIRS, Japan*). Such period usually contains about 10 – 20 accelerating gaps (Ng).

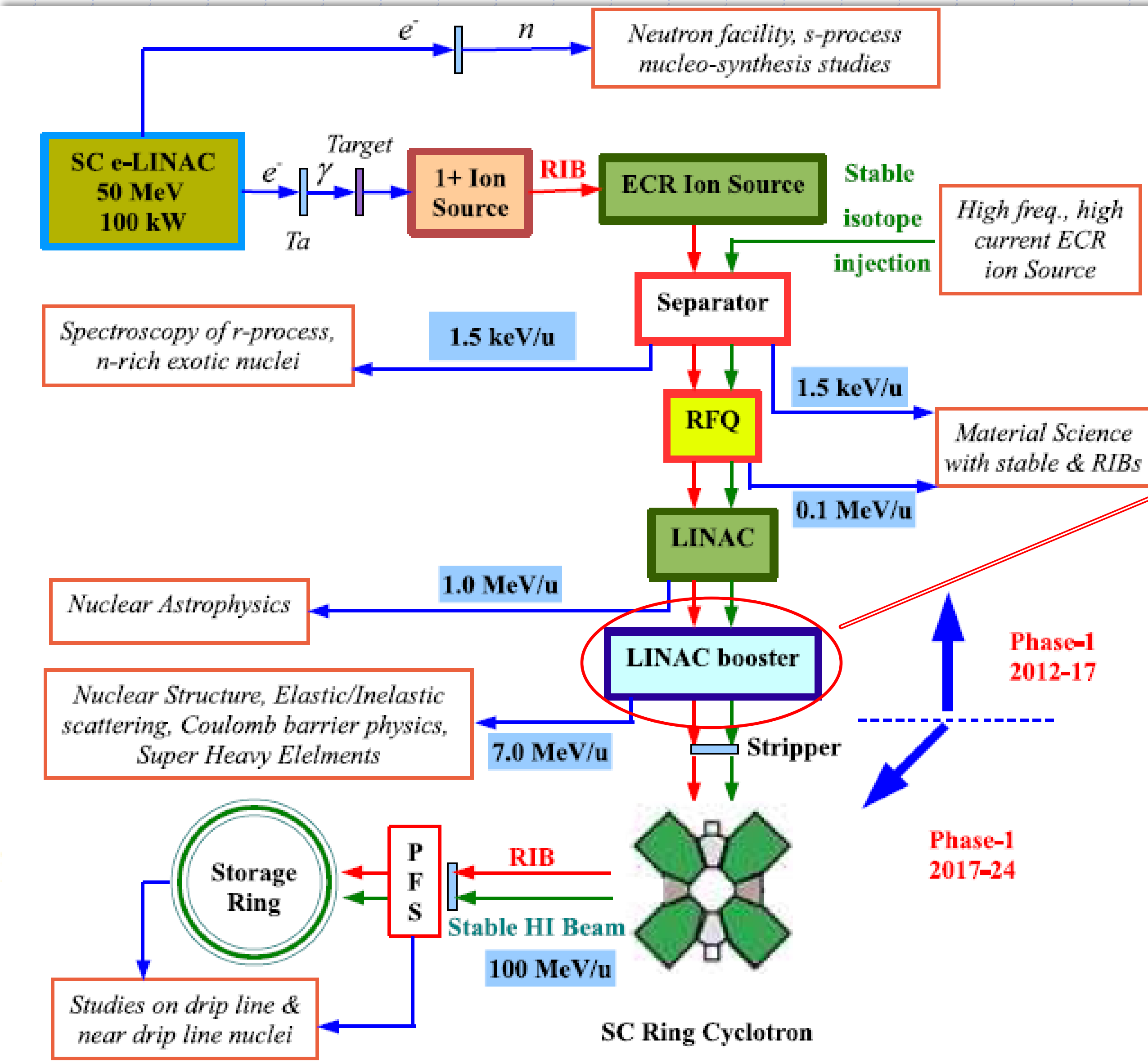
Square Wave Law: Neighboring gaps have same synchronous phase. Every such gap – set can be formed into a separate resonator.



Long multi-gap resonator converted into chain of short independently phased resonators & APF still is as effective as in long structures

S.A.Minaev, Proc. EPAC 1990: Used resonators with 2 gaps
E.S.Masunov et.al , Proc. EPAC 2004: Used resonators with 4 gaps
V.V.Kapin et.al, Proc. RuPAC 2010: Feasibility study of APF realised in short independent resonator using stability diagram. Designed 0.5 MeV/u to 6 MeV/u, q/A =1/8. Less number of independent resonators were required as compared to studies carried out earlier.

PROPOSED ANURIB FACILITY vis-à-vis APF



Goal

APF structure design in series of a double gap quarter wave resonators to be used as SC Linac Booster

Energy Gain: 1.3 MeV/u to ~ 7 MeV/u for ^{238}U

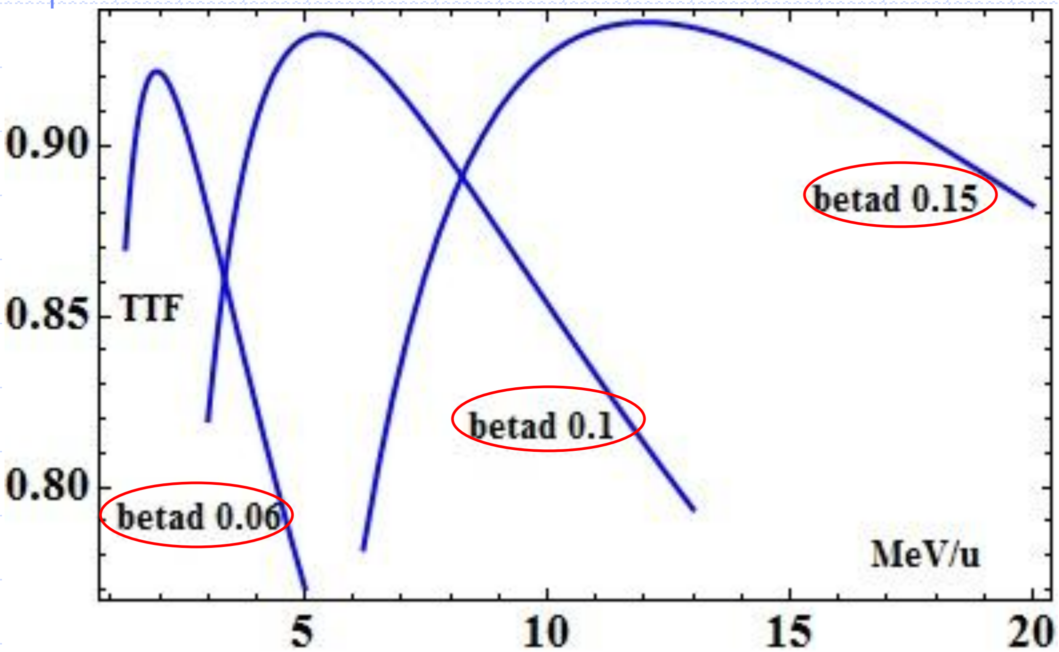
Charge stripping of ^{238}U @ 1.3 MeV/u produces avg. q ~ 36: q/A~ 1/8

Design of A-APF Configuration

Selection of designed beta for QWRs

Desired energy gain
For q/A ~ 1/8 : 1.3 MeV/u to 7 MeV/u
^{16}O (qavg ~ 6 after charge stripping @ 1.3 MeV/u): 1.3 MeV/u to 18 MeV/u
(keeping energy gain per unit charge state same)

Designed beta needs to be chosen to have TTF ~ 0.8 over 1.3 MeV/u to 18 MeV/u range.



TTF of double gap QWR resonating at 100 MHz, aperture dia ~ 20mm & g/βλ ratio ~ 0.2 calculated using formula described in *[J. R. Delayen, NIM 212 (1983) 73-79]*

Stability Analysis (Smith- Gluckstern diagram)

Step wise reference phase oscillation over one period: $\phi_S(\tau) = \bar{\phi} + \tilde{\phi}(\tau), \tau = z / L_f$

L_f being the focusing period length over which the phase excursion completes one cycle. Variable part of above function is constant within two accelerating gaps of a QWR.

Mathieu Hill Equations in terms of phase deviation $\psi = \phi - \phi_S$ and dimensionless radial parameter $\rho = r / L_f$

$$\frac{d^2\psi}{d\tau^2} + P_\psi(\tau).\psi = 0 \; ; P_\psi(\tau) = 2B.\sin[\bar{\phi} + \tilde{\phi}(\tau)] \qquad P_\rho(\tau) \Rightarrow P_\rho(\tau+1)$$

$$\frac{d^2\rho}{d\tau^2} + P_\rho(\tau).\rho = 0 \; ; P_\rho(\tau) = -B.\sin[\bar{\phi} + \tilde{\phi}(\tau) + \psi] \qquad P_\psi(\tau) \Rightarrow P_\psi(\tau+1)$$

with B as focusing parameter given by

$$B = (\pi q E_m / A m_0 c^2)(L_f / \beta_s \lambda)^2 (1 - \beta_s^2)^{3/2}$$

Solution to such equation can be carried out by employing well known matrix multiplication technique.

Creating Smith- Gluckstern diagram

Particular focusing period consisting of say N number of QWRs obeying relation for synchronous phase as $\overline{\phi} + \phi_0 \sin(\tau)$. Different random sets of $\overline{\phi}$ & ϕ_0 are considered. Distance between QWRs & space for a solenoid in each focusing period have been kept.

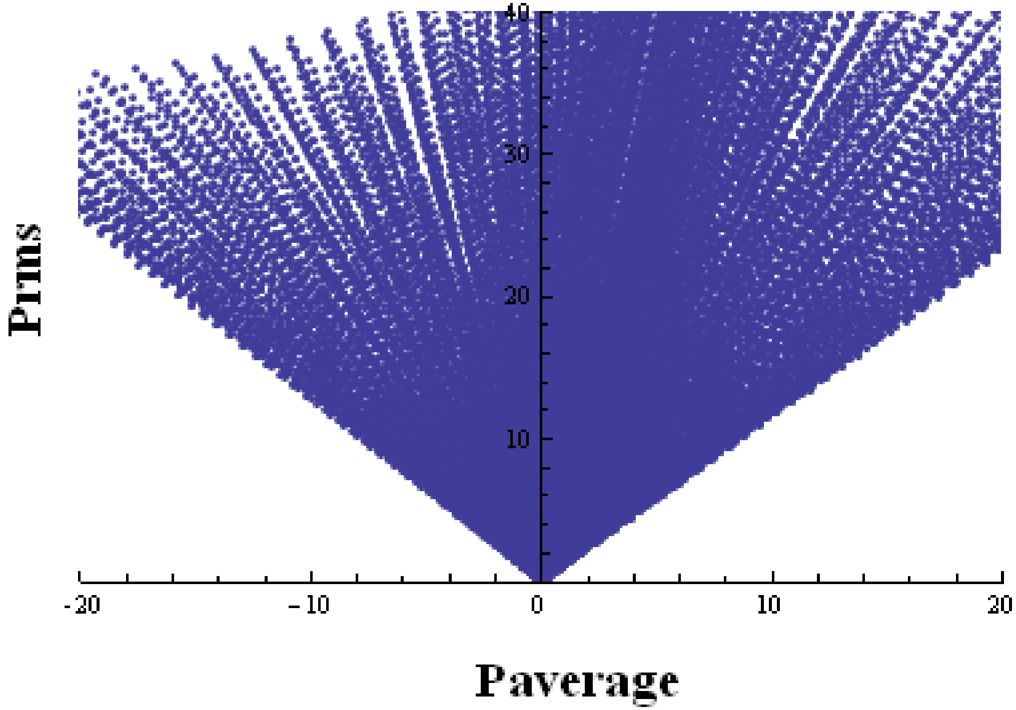
Electric field for a particular set of such phase has been kept same for all the QWR's in a focusing period. Max. Ea considered is 6 MV/m. For each such set of phase variation & electric field RMS & average value of $P_\psi(\tau)$ is calculated creating a point in the space constituted by RMS and average value .

Matrices are obtained by multiplying in proper order the matrices of drift lengths and electric field gap using MATHEMATICA. Transverse (μ_T) and longitudinal phase advance (μ_L) have been calculated using matrix multiplication technique corresponding to each such set (points created in above phase space).

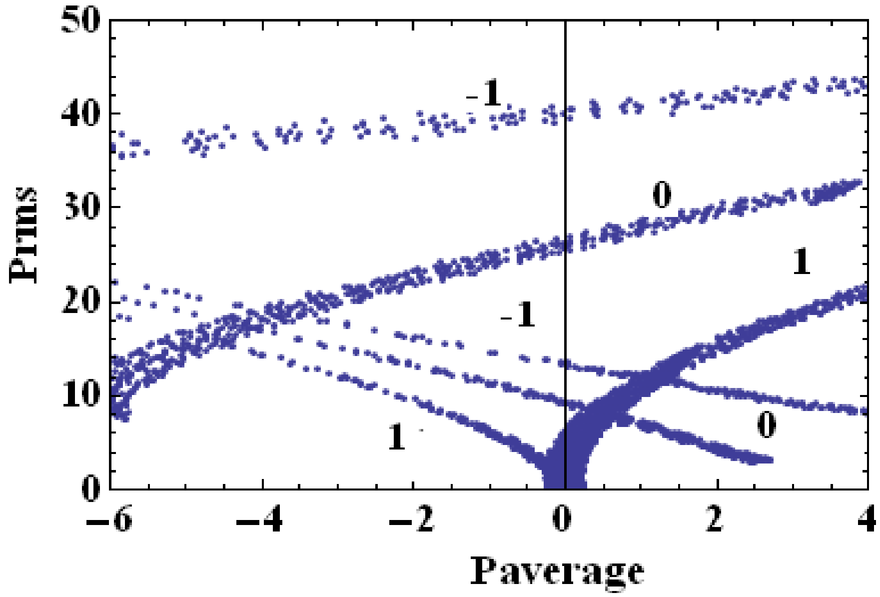
Contours of Cos(μ_L) and Cos(μ_T) having values 1, 0 and -1 are drawn . To ensure stability the operating point is chosen at centre of stability diagram.

Smith- Gluckstern Stability Diagram using MATHEMATICA

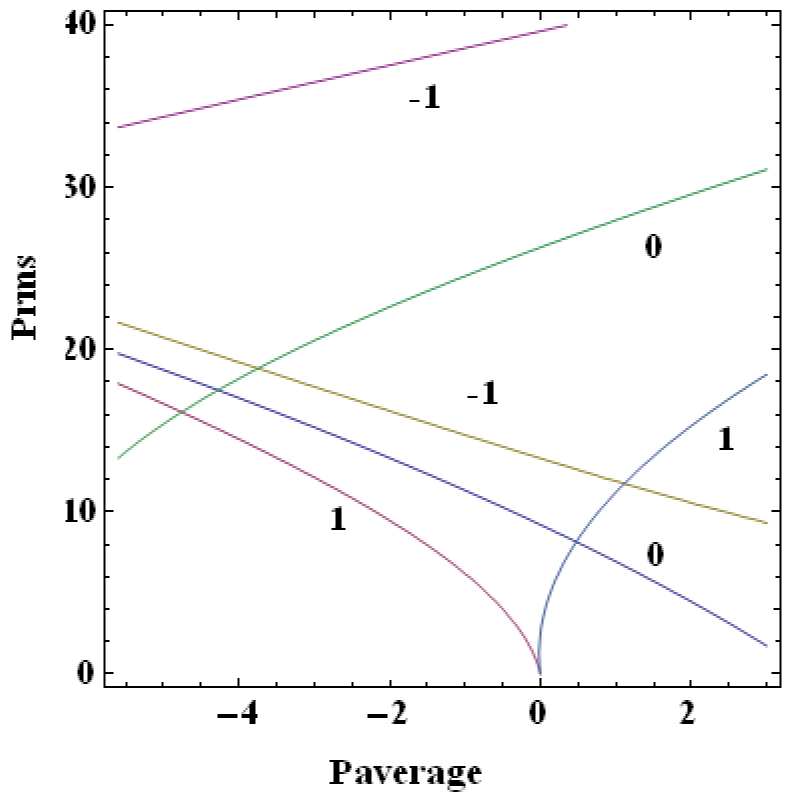
Populating phase space with random sets of $\bar{\phi}$, ϕ_0 & Ea



Smith- Gluckstern Stability Diagram using MATHEMATICA

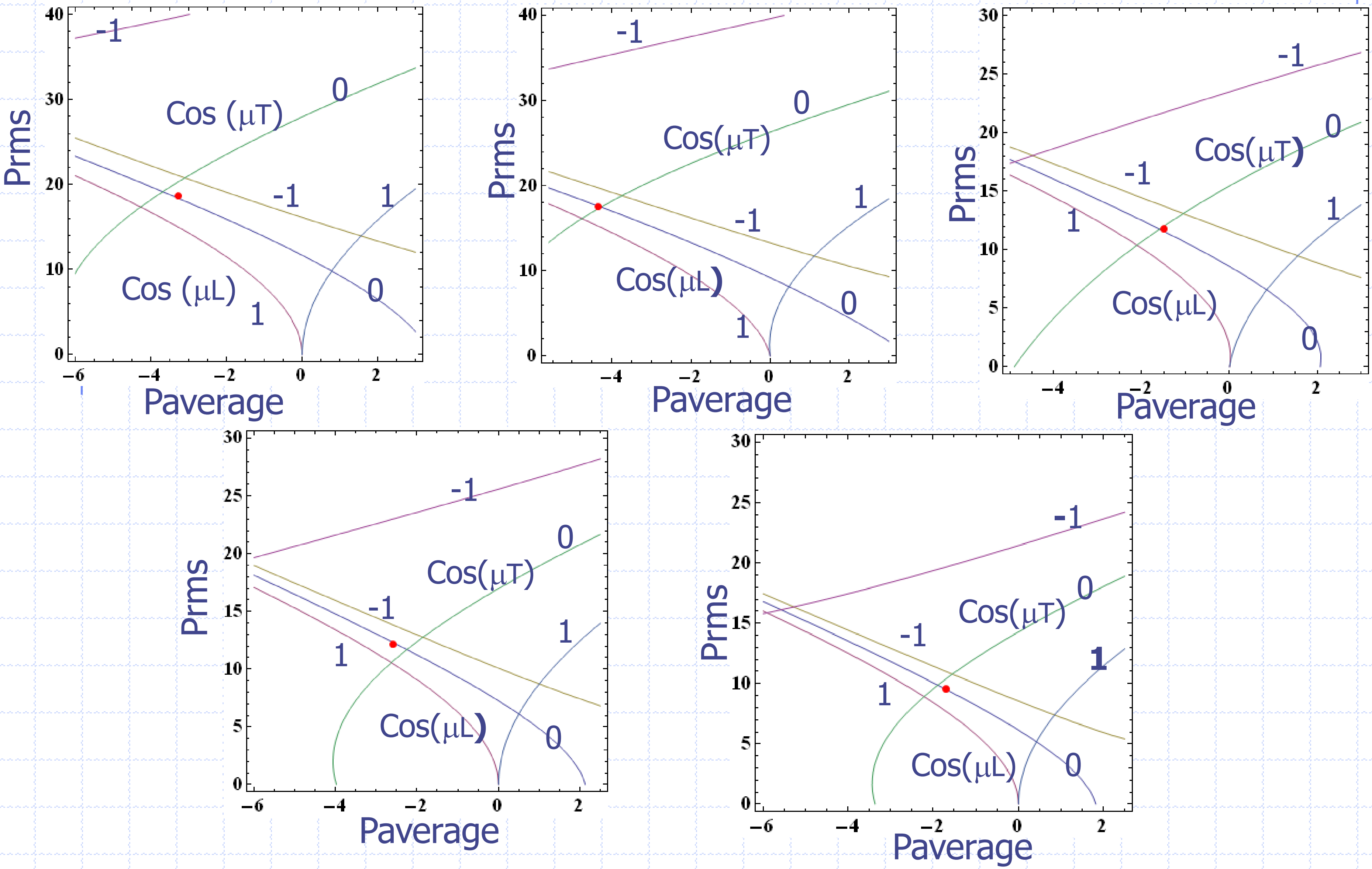


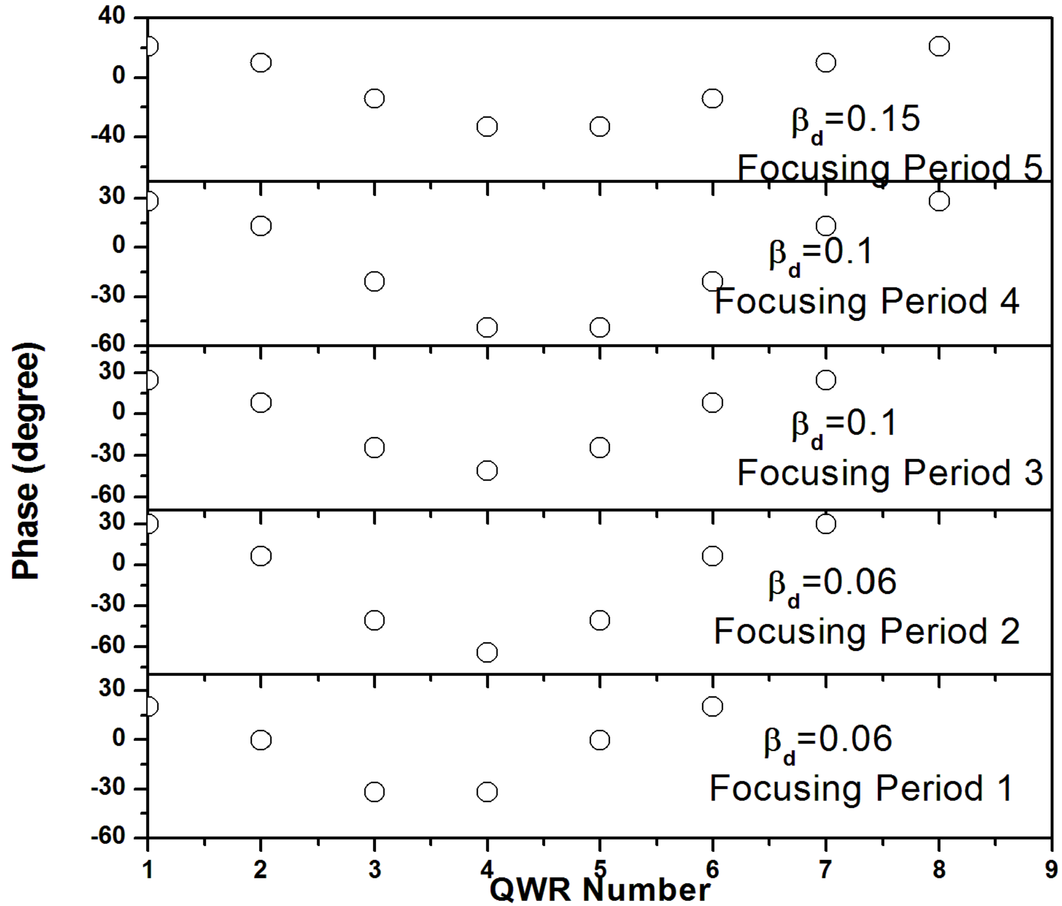
Smith- Gluckstern Stability Diagram using MATHEMATICA



Results of stability Analysis

Optimized design consist of five focusing periods.





Focusing Period #	Energy Input/output (MeV/u)	L (m)	No. of QWR's& β_d
1	1.3-2.06	2.1	6/ 0.06
2	2.06-2.78	2.4	7/0.06
3	2.78 – 3.84	3.24	7/0.1
4	3.84 -5.08	3.66	8/0.1
5	5.08 - 7	4.86	8/0.15

Evaluation of Longitudinal acceptance for focusing period

Smooth approximation with acceleration (since we have SC Linac) has been applied *[J. Qiang et.al, Nucl. Instrum & Methods A 496 (2003) 33]*

Equation of motion in longitudinal dimension

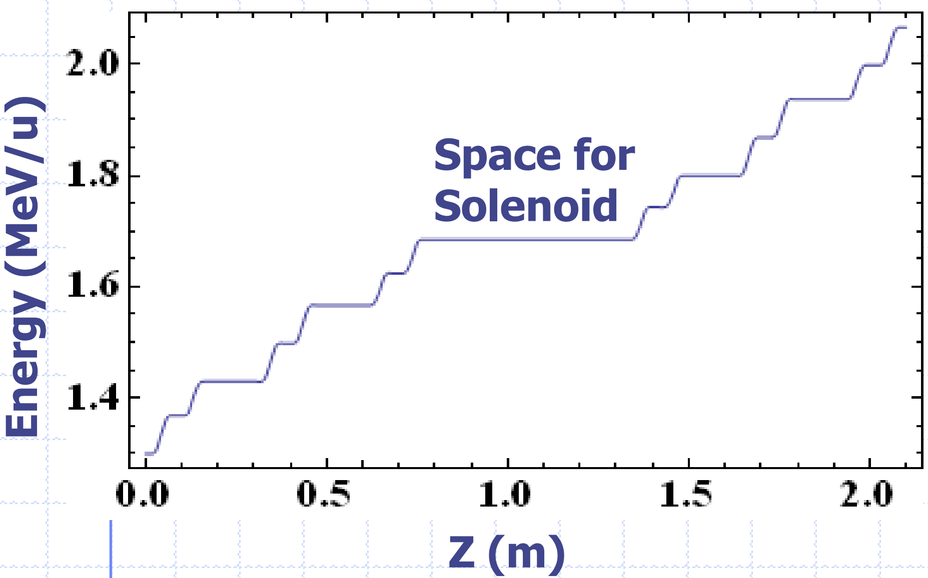
$$\psi'' - \gamma_0^3\beta_0^3(1/\gamma_0^3\beta_0^3)'\psi' - (\omega/c)(1/\gamma_0^3\beta_0^3)(q/Amc^2)\sum_i E_{0i}(\cos(\omega t_o(z)+\theta_i) - \cos(\omega t_0(z)+\theta_i+\psi)) = 0$$

E_{oi} and θ_i denotes amplitude and phase for i[th] cavity. Summation includes all resonators in a focusing period. γ and β are function of longitudinal co-ordinate. Solving the following longitudinal equations one can find the z dependence

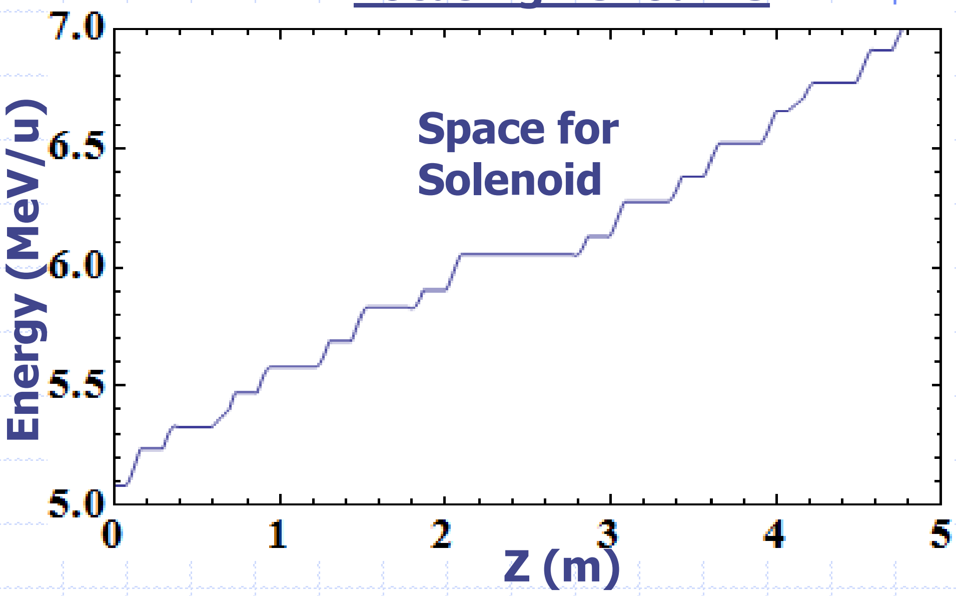
$$\gamma'(z) = (q/Amc^2)\sum_i E_{0i}(\cos(\omega t_o(z)+\theta_i),$$

$$t'(z) = 1/(c\sqrt{1-\gamma^{-2}(z)})$$

Focusing Period #1

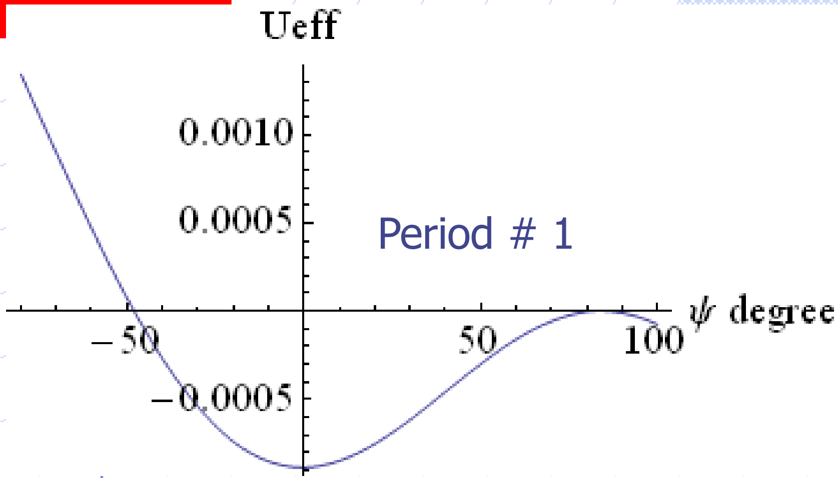


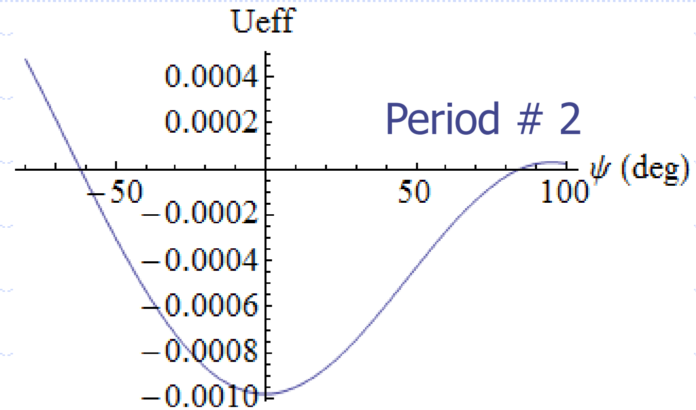
Focusing Period #5

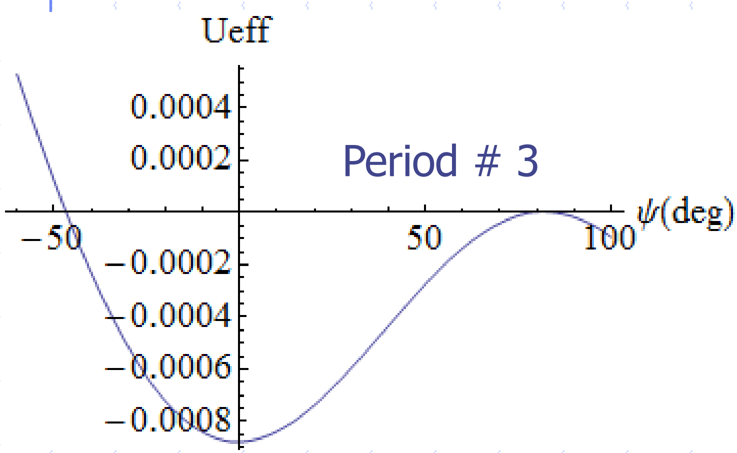


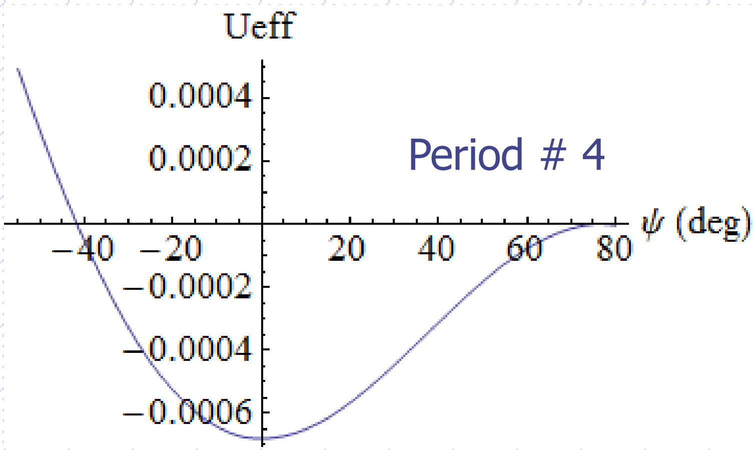
Defining $\varphi = \sqrt{\gamma_0^3 \beta_0^3}\psi$ one can re-write the equation as $\varphi^{//} = F(\varphi, z)$. Net force can be seperated in two parts one for fast oscillation ($\tilde{\phi}$)and other for slow smooth variable ($\bar{\phi}$) with the condition ($|\tilde{\phi}| << |\bar{\phi}|$). So equation of motion can be for slow smooth varying part $\bar{\varphi}^{//} = \bar{F}(\bar{\varphi}) + f(\bar{\varphi})$ and effective potential as

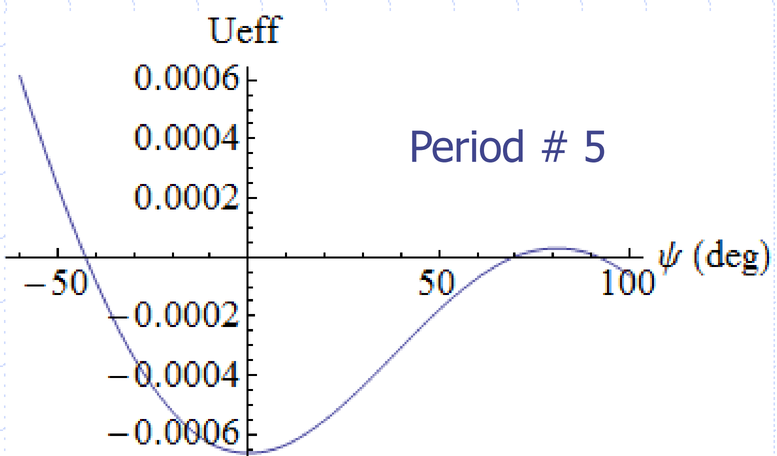
$$U_{eff}(\bar{\varphi}) = -\int_0^{\bar{\varphi}} dx(\bar{F}(x) + f(x))$$


Ueff
0.0010
0.0005
Period # 1
ψ degree
−50
50
100
−0.0005


Ueff
0.0004
0.0002
Period # 2
ψ (deg)
−50
50
100
−0.0002
−0.0004
−0.0006
−0.0008
−0.0010


Ueff
0.0004
0.0002
Period # 3
ψ(deg)
−50
50
100
−0.0002
−0.0004
−0.0006
−0.0008


Ueff
0.0004
0.0002
Period # 4
ψ (deg)
−40
−20
20
40
60
80
−0.0002
−0.0004
−0.0006


Ueff
0.0006
0.0004
0.0002
Period # 5
ψ (deg)
−50
50
100
−0.0002
−0.0004
−0.0006

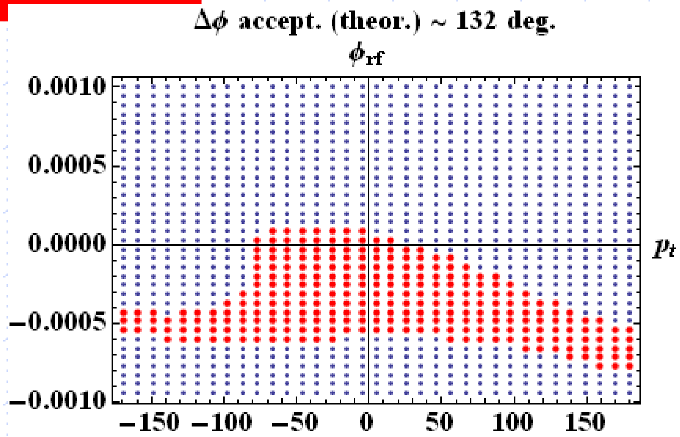
The phase acceptance and the energy width calculated from effective potential are

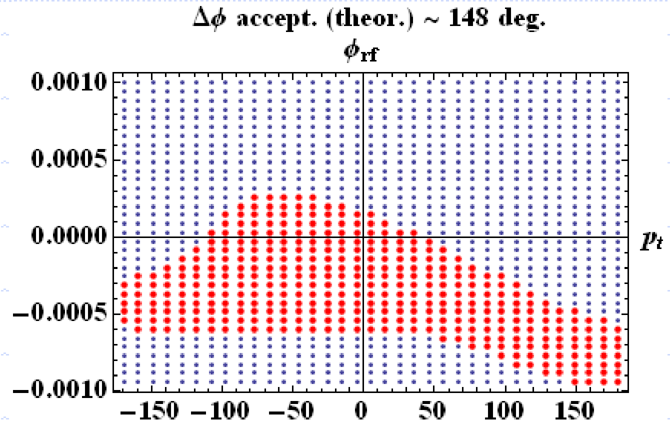
Focusing Period Number	1	2	3	4	5
Phase Acceptance (deg)	131	158	117	124	128
Energy Width (keV/u)	227	338	353	444	631

Solving the following longitudinal equation of motion as described by the set of differential equations mentioned below with different initial conditions of t[z=0] and γ[z=0], one can also determine longitudinal acceptance.

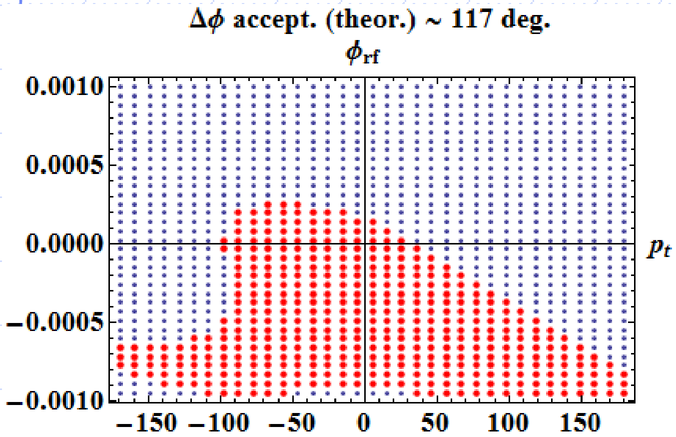
$$\gamma'(z) = \left(\frac{q}{Amc^2}\right)\sum_i E_{0i}(\cos(\omega t_o(z)+\theta_i),$$

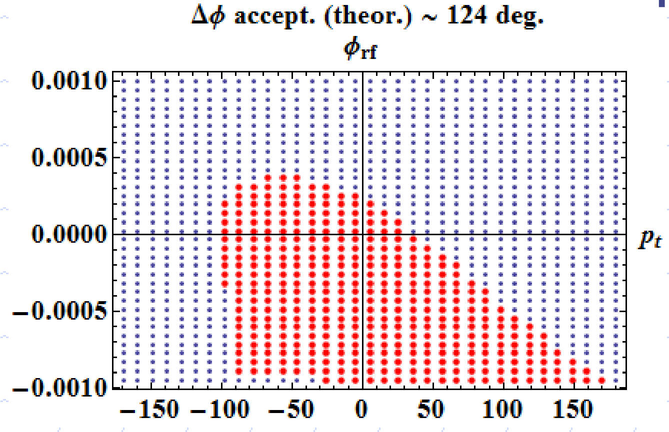
$$t'(z) = 1/(c\sqrt{1-\gamma^{-2}(z)})$$

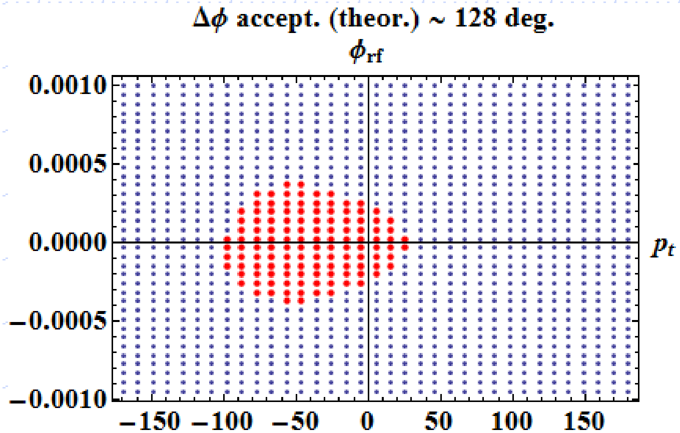




$p_t = \gamma - \gamma_0$



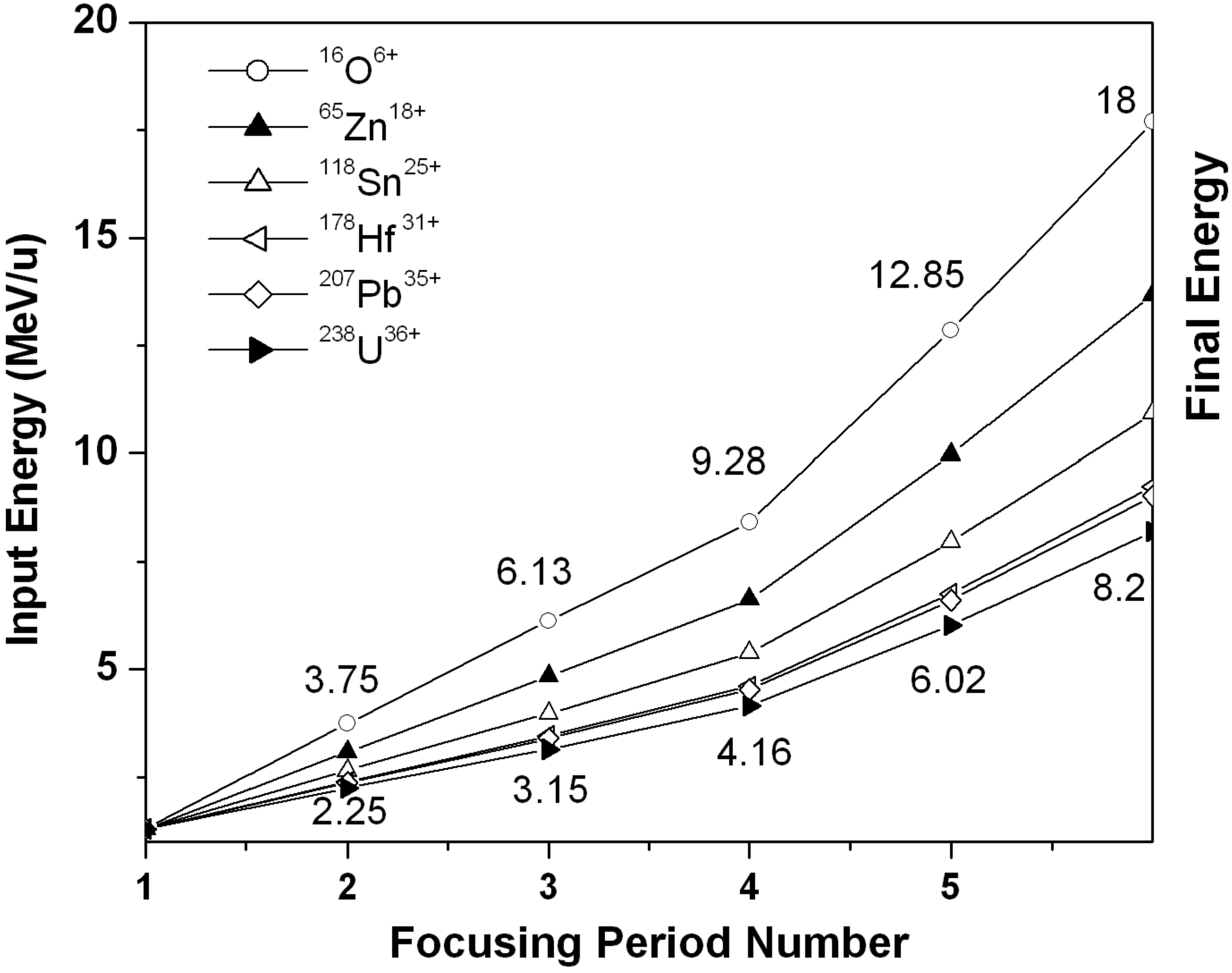




Red dots showing those initial values of pt and Ψ for which final

$$\left| p_t / \gamma_0 \right| < 0.001$$

Energy of different q/A ratio



CONCLUSION

A-APF structure finds its suitability for providing both longitudinal and transverse stability and have appreciable phase acceptance.

1.3 MeV/u to 7 MeV/u (q/A~ 1/8) SC Linac booster have been designed invoking the above advantages. Five such focusing periods are constituted with QWR of designed beta 0.06, 0.1 and 0.15 with a reasonable electric field gradient.

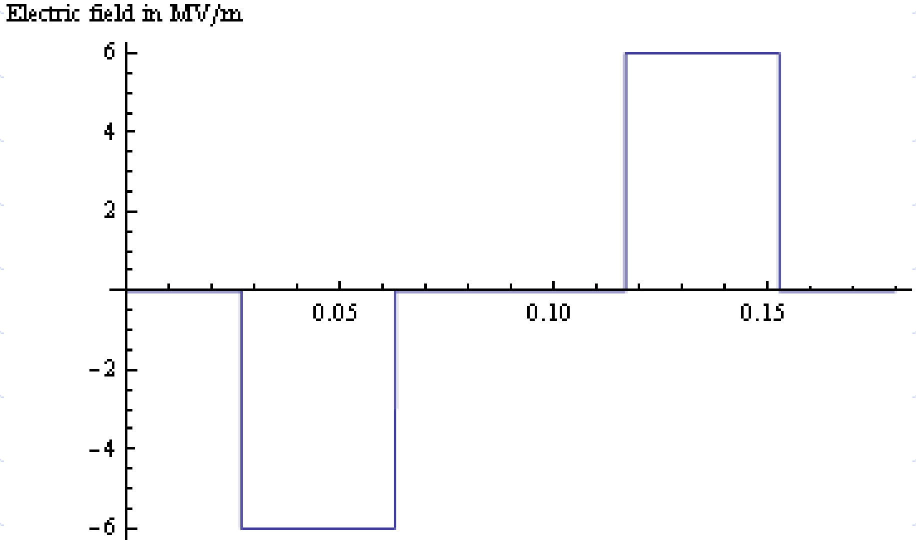
FUTURE WORK

Re-visit the entire beam line consisting of five periods with particle tracking code (such as TRACK or ASTRA) using CST MWS simulated profile for QWRs.

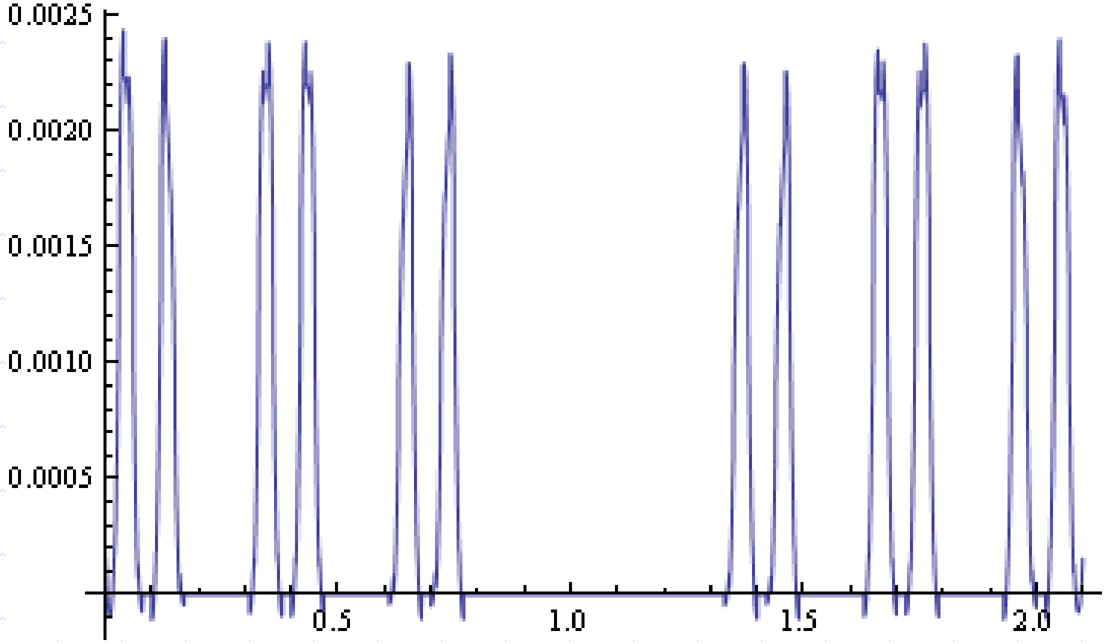
ACKNOWLEDGEMENT

Dr. Alok Chakrabarti for ideas, scientific discussions & useful comments

Electric field of QWR



Electric field seen by synchronous particle: Focusing Period # 2



Simultaneous Acceleration of Multiply Charged Ions through a Superconducting Linac

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(Received 8 November 2000)

The possibility that a linear accelerator could accelerate particles with different charges simultaneously has been known for some time. Notably, proton linacs have accelerated both positive and negative hydrogen ions using the change in the sign of the rf accelerating field when 180° out of phase [1]. With superconducting linacs, where the phase of each cavity is controlled separately, this concept can be generalized to accelerate a range of charge states with the same mass, provided the phases of the bunches can be controlled precisely. This concept can enhance the

The spread in charge states that can be accepted for acceleration [3] depends primarily on the extent that the focusing system can limit emittance growth in transverse phase space. Consequently, the tolerable emittance growth is set by the intensity of lost energetic ions that can produce residual activation of the accelerator components. Therefore, in heavy-ion linacs at low intensity or low energy, a wide range of Δq, about $\pm 10\%$, can be accepted and accelerated. However, in high intensity ($\sim 10^{13}$ uranium nuclei per second) and medium energy ($\sim$400 MeV/u) the tolerable spread of charge states is significantly lower.

Standard periodic focusing theory can be used to analyze the simultaneous acceleration of the several charge states. For example, a spread in charge states of $\pm 2.6\%$ produces a total transverse emittance growth of 6%. This is caused by slightly mismatched conditions for different charge states in the periodic focusing channel with a 60°

TIFR: www.tifr.res.in/~pell/

Average energy gain per cavity 0.4 MV/q

β=0.1, f = 150 MHz

IUAC: www.ivsnet.org/ADS/proceedings/02/I17.pdf

Average energy gain per cavity 0.4 MV/q

β=0.08, f = 97 MHz ($^{12}C^{6+}$ Energy gain 20MeV with 8 resonators)

Present Design

Ea(max) = 6 MV/m, 5.7 MV/m, 4.5 MV/m, 4.8 MV/m, 4.6 MV/m

β=0.06, 0.06, 0.1, 0.1, 0.15

Max energy gain/q:

(0.18 X 6 X 6)+(0.18 X 7 X 5.7)+(0.3 X 7 X 4.5)+(0.3 X 8 X 4.8)
+(0.45 X 8 X 4.6)=51.2 MV/q

Our Case

(7-1.3) X 8 = 45.6 MV/q : Total Energy gain

Avg. Energy gain/cavity: 45.6/38 = 1.2 MV/q

Operating at 89% of maximum possible energy gain