Hadrons at finite temperature

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Topics

- Chiral symmetry of QCD
- Chiral condensate in the medium
- Vector and axial-vector correlators
- Sum rules in the medium
- Spectral function of the ρ meson

- *CBM Physics Book*
- *H. Leutwyler, hep-ph/0212325*
- *J. Alam et al Ann. Phys. 286 (2001) 159*
- *R. Rapp et al Adv. Nucl. Phys. 25 (2000) 1*
- *S. Leupold et al IJMPE 19 (2010) 147*
- *R. S. Hayano et al arXiv:0812.1702*

QCD Lagrangian

- Quantum Chromodynamics(QCD) is the theory of strong interactions

$$\mathcal{L}_{QCD} = \sum_{f=u,d,\cdots} \overline{\psi}_f (i\gamma^\mu D_\mu - m_f)\psi_f - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \; ; \qquad f = 1, N_f$$

  - covariant derivative $\quad D_\mu = \partial_\mu - ig_s \frac{\lambda_a}{2} A^a_\mu \qquad a = 1, 8$
  - gluon field strength tensor

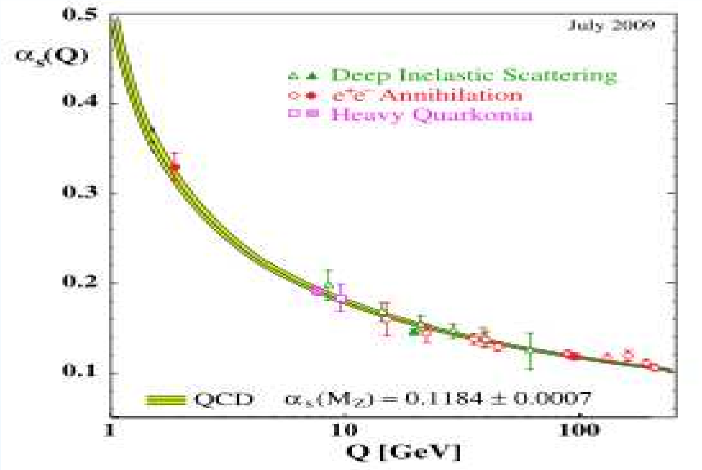
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \; ; \quad f^{abc} \rightarrow SU(3)_c \text{ structure constants}$$

- Due to quantum effects (loops) the coupling $\alpha_s = g_s^2/4\pi$ 'runs' with momentum transfer Q

$$\alpha_s(Q) = \frac{12\pi}{(33 - 2N_f)\ln(\frac{Q^2}{\Lambda^2})}$$

- Renormalisation introduces the QCD scale parameter, $\Lambda \sim 200$ MeV

'running' coupling



- $Q^2 \sim \Lambda^2$
  - q's and g's confined within hadrons $\Rightarrow$ degrees of freedom change to π, p, n etc.
  - perturbative QCD does not work
  - Lattice simulation (LQCD)
  - Effective methods based on symmetries of QCD $\longrightarrow$ Chiral symmetry

- $Q^2 >> \Lambda^2$
  - q's and g's essentially free
  - perturbative region
  - QCD well tested in DIS, jet production etc.

Chiral symmetry

- Consider $\mathcal{L}_{QCD}$ for two massless light flavours u and d

- In terms of left and right handed fields $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\psi = \begin{pmatrix} u_{R,L} \\ d_{R,L} \end{pmatrix}$

$$\mathcal{L}_{QCD} = i\bar{\psi}_R\gamma^\mu D_\mu\psi_R + i\bar{\psi}_L\gamma^\mu D_\mu\psi_L - \frac{1}{4}G^c_{\mu\nu}G^{\mu\nu}_c$$

- $\mathcal{L}_{QCD}$ is invariant under chiral transformations i.e. separate flavour transformations on left and right components of u and d

$$\psi_R \to U_R\psi_R \qquad U_R = e^{i\alpha^a_R\tau^a/2} \in SU(2)_R \quad a = 1,2,3$$
$$\psi_L \to U_L\psi_L \qquad U_L = e^{i\alpha^a_L\tau^a/2} \in SU(2)_L$$

- Under this global $SU(2)_R \times SU(2)_L$ symmetry, the conserved currents are

$$j_R^{\mu\ a} = \bar{\psi}_R\gamma^\mu\frac{\tau_a}{2}\psi_R \quad \& \quad j_L^{\mu\ a} = \bar{\psi}_L\gamma^\mu\frac{\tau_a}{2}\psi_L \quad \text{with} \quad \partial_\mu j_R^{\mu\ a} = \partial_\mu j_L^{\mu\ a} = 0$$

Chiral symmetry

- Thus chiral symmetry of $\mathcal{L}_{QCD} \Rightarrow$ left and right handed quarks do not communicate $\Rightarrow$ 'handedness' is preserved in dynamical processes
- A mass term '$m_f(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$' allows for $L \leftrightarrow R$ transitions; chiral limit$\Rightarrow m_f = 0$
- chiral currents can be expressed in terms of conserved vector and axialvector currents

$$\begin{aligned} j_V^{\mu\ a} &= j_R^{\mu\ a} + j_L^{\mu\ a} = \bar{\psi}\gamma^\mu \frac{\tau_a}{2}\psi \\ j_A^{\mu\ a} &= j_R^{\mu\ a} - j_L^{\mu\ a} = \bar{\psi}\gamma^\mu\gamma^5 \frac{\tau_a}{2}\psi \end{aligned}$$

- The corresponding charges generate the algebra of $SU(2)_V$ and $SU(2)_A$

$$Q_V^a = \int d^3x\, j_V^{0\ a}(x) \quad \text{and} \quad Q_A^a = \int d^3x\, j_A^{0\ a}(x)$$

- They commute with the QCD Hamiltonian

$$[Q_V^a, H_{QCD}^{m_f=0}] = 0 \quad \text{and} \quad [Q_A^a, H_{QCD}^{m_f=0}] = 0$$

Chiral symmetry

- So $\mathcal{L}_{QCD}$ in the limit of massless quarks has global chiral symmetry
  What about the vacuum (ground state) of QCD ?

- Essential criteria for a symmetry to be realised in terms of degenerate multiplets is:
  $U_{sym}|0\rangle = |0\rangle$ ground state is invariant under symmetry transformation
  $Q_{sym}|0\rangle = 0$ symmetry charges annihilate the vacuum

- This is the (normal) Wigner-Weyl mode of realisation of symmetry

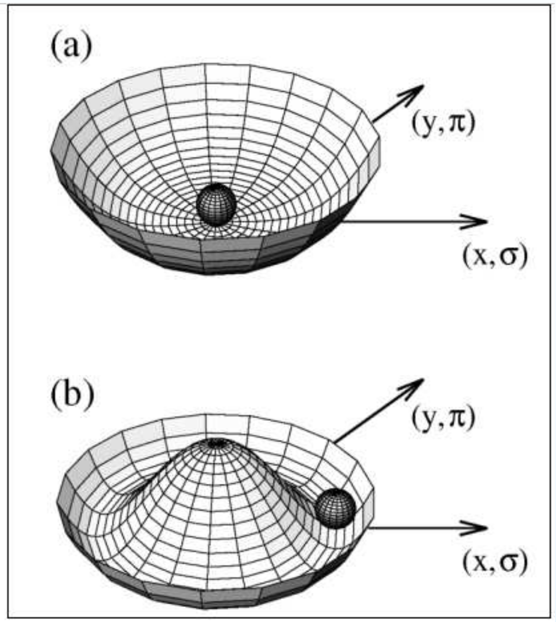
- For the vector charges, $Q_V^a|0\rangle = 0$ *Wafa & Witten NPB (1984)*
  $\Longrightarrow$ vacuum is symmetric under $SU(2)_V$; isospin singlet
  $\Longrightarrow$ 'degenerate' (isospin) doublet n, p triplets ρ^+, ρ^0, ρ^- etc.
  $\Longrightarrow$ operators generate transition within multiplets e.g. $\tau^+|n\rangle = |p\rangle$ etc.

- But for the three axial charges, we can have two possibilities

Chiral symmetry

- (a) $Q_A^a|0\rangle = 0$
- $\Rightarrow$ unique vacuum
- degenerate multiplets of opposite parity

- (b) $Q_A^a|0\rangle \neq 0$
- $\Rightarrow$ degenerate vacua
- massless pseudoscalars $\rightarrow$ Goldstone bosons
- spontaneously broken symmetry



- We observe :
  - no degenerate parity partners (~ 600 MeV difference in mass)
    $m_\rho[J^P = 1^-] = 770 \text{ MeV}/m_{a_1}[J^P = 1^+] = 1260 \text{ MeV}$
    $m_N[J^P = 1/2^+] = 940 \text{ MeV}/m_N^*[J^P = 1/2^-] = 1535 \text{ MeV}$ etc.
  - triplet of 'light' pions $\Rightarrow$ Goldstone bosons
- Assume : $SU(2)_R \times SU(2)_L$ sp. broken to $SU(2)_V$

Chiral condensate

- For any operator P, if $\langle 0|[Q,P]|0\rangle \neq 0$, this expectation value is an order parameter
- with $P^b = \bar{\psi}\gamma^5\tau^b\psi$, $\quad [Q_A^a, P^b] = -\delta^{ab}\bar{\psi}\psi$
- $Q_A^a|0\rangle \neq 0$ implies $\langle 0|[Q_A^a, P^b]|0\rangle \to \langle 0|\bar{\psi}\psi|0\rangle \neq 0$
  $\Longrightarrow$ chiral condensate is an order parameter for chiral symmetry breaking
- In addition, there is an explicit breaking due to $m_u, m_d \neq 0$
- The symmetry breaking parameters are related to the pion mass through Gell Mann-Oaks-Renner (GOR) relation

$$m_\pi^2 F_\pi^2 = -(m_u + m_d)\langle 0|\bar{\psi}\psi|0\rangle + O(m_{u,d}^2)$$

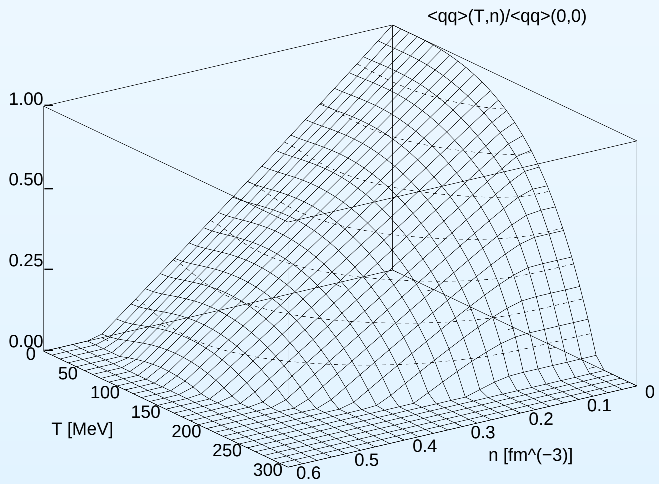
  in the chiral limit ($m_{u,d} = 0$) $m_\pi = 0$
- For $F_\pi = 93$ MeV (from $\pi^+ \to \mu^+\nu_\mu$ decay)
  $\langle 0|\bar{\psi}\psi|0\rangle \sim -(250\ \mathrm{MeV})^3$

Chiral condensate in the medium

- How does the chiral condensate change in a hot/dense medium e.g. in Relativistic Heavy Ion Collisions?
- A first estimate can be obtained from linear density expansions
- approximate the thermal medium by non-interacting light hadrons; pions at finite T and nucleons at finite μ_B

$$\langle \mathcal{O} \rangle = \langle 0|\mathcal{O}|0\rangle + \int \frac{d^3p}{(2\pi)^3 2p_0} n_\pi \langle \pi|\mathcal{O}|\pi\rangle + \int_0^{p_F} \frac{d^3p}{(2\pi)^3 2p_0} \langle N|\mathcal{O}|N\rangle + \cdots$$

  - At lowest order
    $\langle \bar{\psi}\psi \rangle \simeq \langle 0|\bar{\psi}\psi|0\rangle \left(1 - \frac{T^2}{8F_\pi^2} - \frac{\rho_N}{3\rho_0}\right)$
    for $m_\pi = 0$ (chiral limit)
  - this naive estimate predicts chiral symmetry restoration at $T \sim 250$ MeV and/or $\rho_N \sim 3\rho_0$

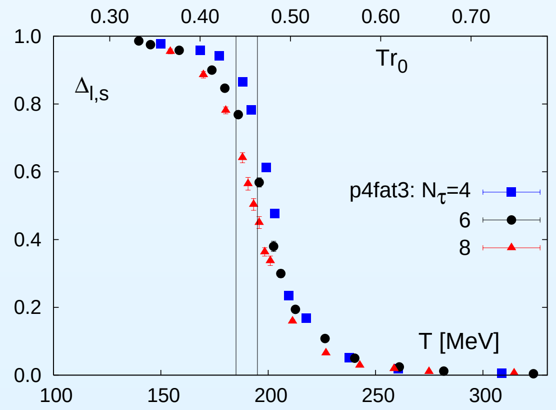


Chiral condensate in medium

- The pion decay constant F_π is also an order parameter for chiral phase transition
- in pionic medium $F_\pi(T) = F_\pi\left(1 - \frac{T^2}{12F_\pi^2}\right) \Rightarrow$ also decreases with T
- Lattice simulations of QCD thermodynamics:

$$\langle\bar{\psi}\psi\rangle_T \sim \frac{\partial P(T,V)}{\partial m_q} \quad \text{where} \quad P = T\frac{\partial}{\partial V}\ln\mathcal{Z}$$

- The chiral condensate shows a rapid drop in the transition region
  - For $T > T_c$, the condensate eventually disappears $\Longrightarrow$ chiral symmetry is realised in the Wigner-Weyl mode
  - For three quark flavours the chiral transition is expected to be of second order



A. Bazavov et al PRD80 (2009) 014504

Current Correlators

- The chiral condensate is not an experimentally measurable quantity
- Current correlators provide an useful framework to connect QCD with observables (hadrons)
- These are expectation values of two-point functions of (local) currents
- Consider the correlators of vector currents (j^μ_V) and axial-vector currents (j^μ_A) of QCD

$$\Pi^{\mu\nu}_V(q) = i\int d^4x e^{iq\cdot x}\langle 0|Tj^\mu_V(x)j^\nu_V(0)|0\rangle$$
$$\Pi^{\mu\nu}_A(q) = i\int d^4x e^{iq\cdot x}\langle 0|Tj^\mu_A(x)j^\nu_A(0)|0\rangle$$

- $\mathrm{Im}\Pi$ contains the spectral information $\longrightarrow$ spectral density
- The currents (j^μ) couple to individual hadrons as well as multi-particle states with the same q. nos. $\Rightarrow$ spectral densities contain peak and continuum structure

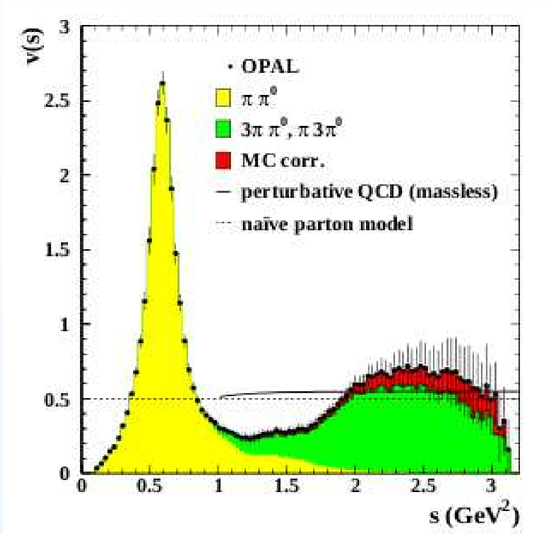
Current Correlators in vacuum

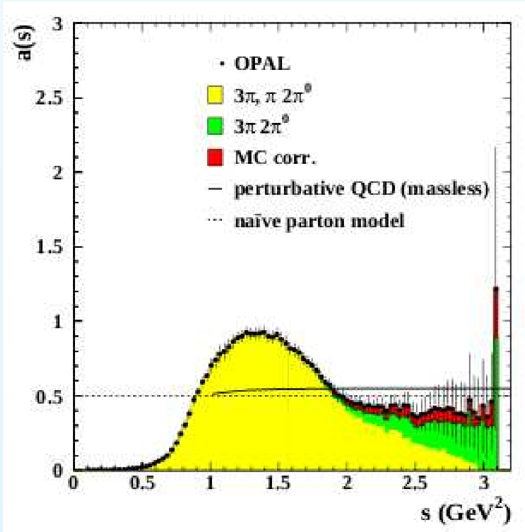
- The V and A correlators are identical to all orders in perturbation theory
  Chiral symmetry implies : $\mathrm{Im}\Pi_V(q) = \mathrm{Im}\Pi_A(q)$

- $\mathrm{Im}\Pi_V$ and $\mathrm{Im}\Pi_A$ have been measured at LEP in τ decays into even and odd number of pions $[\tau \longrightarrow \nu_\tau + n\pi]$ by ALEPH and OPAL Collaborations

- the quantum numbers of $\vec{j}^{\mu}_V$ $[I^G(J^P) = 1^+(1^-)]$ and $\vec{j}^{\mu}_A$ $[I^G(J^P) = 1^-(1^+)]$ coincide with those of ρ and a_1 mesons - peaks dominate at low q^2

- very different spectral shape $\Rightarrow$ broken chiral symmetry in vacuum





EPJC 7 (1999) 571

Current Correlators in medium

- In the strongly interacting medium the spectral density may change; peaks could become broader and pole positions may shift
- $\mathrm{Im}\Pi_V$ is accessible through EM probes in particular, the dilepton spectra from heavy ion collisions
- However, it is difficult to measure $\mathrm{Im}\Pi_A$ in the medium
  $\Rightarrow$ final state interactions would modify the signal in the $\pi^{\pm}\gamma$ or 3π invariant mass spectra
- Since a simultaneous measurement does not appear to be feasible it is essential to put constraints on spectral densities
- QCD Sum Rules are useful for this purpose
- In addition, spectral densities can be related to chiral order parameters through Weinberg Sum Rules

QCD Sum Rules

- Hadron properties can be obtained in terms of QCD parameters through QCD Sum Rules *M. Shifman et al NPB 147 (1979) 385*

- Using analyticity a dispersion relation is written for the correlation function

$$\Pi(q) = \frac{1}{\pi}\int \frac{\mathrm{Im}\Pi^{had}(s)}{(s-q^2)}ds \quad + \ \mathrm{subtractions}$$

- $\Pi(q)$ is also obtained using Operator Product Expansion (for $Q^2 = -q^2 >> 0$)

- Matching the two expressions of $\Pi(q)$ for large space-like momenta $\Rightarrow$ Sum Rules

- In OPE, the correlator is expanded in terms of local operators composed of quark and gluon fields of increasing dimension

$$i\int d^4x e^{iq\cdot x} T[j(x)j(0)] \overset{large\ Q^2}{\longrightarrow} C_1 I + \sum_n C_n(q)\mathcal{O}_n$$

- $C_n \rightarrow$ Wilson coefficients (can be found by taking appropriate matrix elements on both sides)

QCD Sum Rules

- The coefficients C_n fall off as powers of $1/Q^2 \Longrightarrow$ lower dimensional operators e.g. $m_q\bar{\psi}\psi$, $G_{\mu\nu}G^{\mu\nu}$, $\bar{\psi}\Gamma\psi\bar{\psi}\Gamma\psi$ dominate the sum rule

- expectation values of these operators provide non-perturbative contributions

- Parametrize the vector spectral density as

$$\mathrm{Im}\Pi_V(s) = 2\pi m_\rho^2 F_\rho^2 \delta(s - m_\rho^2) \quad + \quad \frac{1}{4\pi}\left(1 + \frac{\alpha_s}{\pi}\right)\theta(s - s_{th})$$

$$\text{pole} \quad + \quad \text{continuum}$$

- Vacuum Sum Rule

$$m_\rho^2 F_\rho^2 e^{-m_\rho^2/M^2} - \frac{M^2}{8\pi^2}\left(1 + \frac{\alpha_s}{\pi}\right)(1 - e^{-s_{th}/M^2})$$
$$= \langle 0|m\overline{\psi}\psi|0\rangle + \langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}|0\rangle - \frac{56\alpha_s}{81M^2}\langle 0|4\text{ quark}|0\rangle + \cdots$$

- At $T \neq 0$ there are additional considerations for both the spectral and OPE sides

In-medium correlators (spectral side)

- Lorentz invariance is not manifest due to existence of a preferred frame
  $\Rightarrow \Pi^{\mu\nu}$ become functions of q_0 and $\vec{q}$ separately instead of q^2

- restored by introducing u_μ (the four-velocity of the medium)
  $\Rightarrow q_0$ and $\vec{q}$ can be defined in terms of two scalars
  $\omega = u \cdot q$ [$= q_0$ in the rest frame with $u_\mu = (1,0,0,0)$]
  $\overline{q} = \sqrt{\omega^2 - q^2}$ [$= |\vec{q}|$ in rest frame]

- For $\vec{q} \neq 0$ the correlation function splits into longitudinal and transverse components
  $$\Pi^{\mu\nu}(q_0,\vec{q}) = P^{\mu\nu}\Pi_T(q_0,\vec{q}) + Q^{\mu\nu}\Pi_L(q_0,\vec{q})$$
  where $P^{\mu\nu}$ and $Q^{\mu\nu}$ are the corresponding projection tensors

- So, in the medium we have two components of the correlation function, each a function of two variables

Thermal QCD Sum Rules

- Additional scalar operators emerge from tensors by contracting with the velocity vector u_μ e.g. $u^\mu \Theta_{\mu\nu} u^\nu$ where $\Theta_{\mu\nu}$ is the stress tensor of QCD

- The vacuum condensates to be replaced by in-medium ones
$\langle 0|\mathcal{O}|0\rangle \longrightarrow \langle \mathcal{O}\rangle_T = Tr[e^{-\beta H}\mathcal{O}]/Tr[e^{-\beta H}]$

- one gets two sum rules; longitudinal and transverse *S. Mallik et al PRD58, 096011*

$$
\begin{aligned}
F_\rho^2(T)e^{-m_\rho^2(T)/M^2} + I_L(M^2) &= \frac{M^2}{8\pi^2} + \frac{\langle \mathrm{O}\rangle_T}{M^2} \\
m_\rho^2(T)F_\rho^2(T)e^{-m_\rho^2(T)/M^2} + I_T(M^2) &= \frac{M^4}{8\pi^2} - \langle \mathrm{O}\rangle_T
\end{aligned}
$$

$\langle \mathrm{O}\rangle_T = m\langle\overline{\psi}\psi\rangle_T + \frac{\langle G^2\rangle_T}{24} + \langle \mathrm{new\ \ operators}\rangle$

- calculate the spectral density from an effective theory and use sum rules to constrain parameters

Weinberg Sum Rules

- The difference of the vector and axial vector spectral densities are quantified by the Weinberg Sum Rules

$$\int \frac{ds}{s\pi}[\mathrm{Im}\Pi^V(s) - \mathrm{Im}\Pi^A(s)] = F_\pi^2$$
$$\int \frac{ds}{\pi}[\mathrm{Im}\Pi^V(s) - \mathrm{Im}\Pi^A(s)] = 0$$
$$\int \frac{sds}{\pi}[\mathrm{Im}\Pi^V(s) - \mathrm{Im}\Pi^A(s)] = 2\pi\langle 0|4\ \mathrm{quark}|0\rangle$$

- In thermal medium
  - The integrals become energy (q_0) integrals
  - each sum rule applies for a fixed 3-momentum ($\vec{q}$) and must be obeyed at each value of the momentum
  - the spectral densities split into T and L modes
  - Constrains both energy and momentum dependence of in-medium spectral densities

Correlators & chiral symmetry restoration

- Possible scenarios of approach to chiral symmetry restoration based on Weinberg Sum Rules at $T \neq 0$ *J.I.Kapusta & E. Shuryak, PRD49 (1994) 4694*
  - Thermal pions induce mixing of V and A correlators. To lowest order

$$\begin{aligned} \mathrm{Im}\Pi_V(T) &= [1-\epsilon(T)]\,\mathrm{Im}\Pi_V^{vac} + \epsilon(T)\mathrm{Im}\Pi_A^{vac} \\ \mathrm{Im}\Pi_A(T) &= [1-\epsilon(T)]\,\mathrm{Im}\Pi_A^{vac} + \epsilon(T)\mathrm{Im}\Pi_V^{vac} \end{aligned} \qquad \epsilon = \frac{T^2}{6F_\pi^2}$$

    Maximal mixing $\Rightarrow$ CSR for $\epsilon = \frac{1}{2} \Rightarrow T_c \sim 164$ MeV

  - The peak positions of $\mathrm{Im}\Pi_V$ and $\mathrm{Im}\Pi_A$ may change with T $\Rightarrow$ masses may shift towards each other or go to zero and become degenerate at T_c

  - Close to T_c the self energy of hadrons may increase and resonance structure may become broad and merge with the continuum $\Rightarrow$ a flat spectral shape in both cases

- The sum rules by themselves cannot indicate the preferred scenario

Detecting in-medium correlators

- Approach to CSR involves a reshaping of one or both correlators
- A simultaneous measurement of $\mathrm{Im}\Pi_V$ and $\mathrm{Im}\Pi_A$ is the best way to study CSR
- not possible due to difficulties in measurement of $\mathrm{Im}\Pi_A$
- Consideration of indirect approaches:
  - Theoretical calculation of $\mathrm{Im}\Pi_V$ and $\mathrm{Im}\Pi_A$ correlators involving detailed consideration of many-body effects in a thermal field theoretical framework based on chiral effective interactions
  - Using $\mathrm{Im}\Pi_V$ to evaluate dilepton spectra and compare with data
  - Using the V and A correlators in WSRs to obtain the temperature dependence of order parameters e.g. F_π and 4-quark condensate and compare with LQCD results for those

Dilepton emission rate

- Dilepton emission rate is given by the thermal expectation value of the correlator of EM currents *McLerran & Toimela PRD (1985)*

$$\frac{dN_{l^+l^-}}{d^4x\, d^4q} = -\frac{\alpha^2}{3\pi^3\, q^2}\frac{g^{\mu\nu}}{e^{\beta q_0}+1}\,\mathrm{Im}W_{\mu\nu}(q)$$

$$W_{\mu\nu}(q) = \int d^4x\, e^{iq\cdot x}\langle T\, J_\mu^{em}(x)J_\nu^{em}(0)\rangle_T \qquad J_\mu^{em} = \sum_f e_f\bar\psi_f\gamma_\mu\psi_f$$

- At low invariant mass M, EM current is decomposed into vector currents

$$J_\mu^{em} = \underset{I=1}{J_\mu^\rho} + \underset{I=0}{J_\mu^\omega} + \cdots$$

- Vector currents converted to vector meson fields (VMD) e.g. $J_\mu^\rho = F_\rho m_\rho \rho_\mu$

$$\mathrm{Im}W^{\mu\nu} \longrightarrow \sum_{V=\rho,\omega,\phi}\mathrm{Im}\Pi_V^{\mu\nu} \longrightarrow \sum_{V=\rho,\omega,\phi}\mathrm{Im}D_V^{\mu\nu}$$

- the essential quantity is the imaginary part of the in-medium vector propagator D_V

ρ spectral function

- The full propagator is obtained through a Dyson equation

$$D = D^0 + D^0\Sigma\, D^0 + D^0\Sigma\, D^0\Sigma\, D^0 + \cdots .$$
$$= \frac{D^0}{1+\Sigma D^0} = \frac{1}{p^2 - m^2 + \Sigma}$$

- spectral function

$$A = \mathrm{Im}D = \frac{\mathrm{Im}\Sigma}{(p^2 - m^2 + \mathrm{Re}\Sigma)^2 + (\mathrm{Im}\Sigma)^2}$$

- Real part gives pole shift & Imaginary part leads to broadening

- For ρ meson (spin 1) $\Sigma^{\mu\nu} = P^{\mu\nu}\Sigma_t + Q^{\mu\nu}\Sigma_l$

  from which we get $\Sigma_l = \frac{\Sigma^{00}}{\vec{q}^2}$ and $\Sigma_t = -\frac{1}{2}(\Sigma^\mu_\mu + q^2\Sigma_l)$

- Spin averaged spectral function: $A_\rho = \frac{1}{3}[2A^t_\rho + A^l_\rho]$

ρ self energy

- Essential quantity to find is the ρ self-energy Σ_ρ in the medium
  - Linear density approximation

$$\Sigma_\rho(q) = \sum_h \int \frac{d^3p}{(2\pi)^3} f_h(p) T_{h\rho}(p,q) \rightarrow \sum_h n_h T_{h\rho}$$

$T_{h\rho} \rightarrow$ forward scattering amplitude ($h = \pi, N$) *Eletsky et al PRC (2001)*

  - Field Theoretic approach using chiral effective interactions
    - Massive Yang-Mills *Song et al PRD (1996)*
    - Hidden Local Symmetry *Bando et al PRL (1985)*
    - Chiral Perturbation Theory with massive spin-1 fields *Ecker et al PLB (1989)*
    - These approaches start with chiral pion Lagrangians and introduce vector meson fields through 'gauging'

Chiral effective theory (pions)

- The low energy effective theory of QCD is constructed in terms of fields of observed particles by utilizing the underlying chiral symmetry
- First determine how Goldstone and non-Goldstone fields transform under chiral transformations
- All terms built out of the observed fields and invariant under these transformation rules form a piece in $\mathcal{L}_{eff}$
- The Goldstone bosons (pions) are collected in a matrix $U(x) = exp[i\tau_a \pi_a(x)/F_\pi]$ which transforms as

$$U'(x) = g_R U(x) g_L^\dagger \qquad g_{R,L} \in SU(2)_{R,L}$$

- $\mathcal{L}_{eff} = \mathcal{L}_{eff}(U, \partial U, \partial^2 U \cdots)$ ordered in increasing number of derivatives of $U(x)$
- The leading term involves two derivatives in U

$$\mathcal{L}_{eff}^{(2)} = \frac{F_\pi^2}{4} Tr[\partial_\mu U^\dagger \partial^\mu U]$$

H. Leutwyler, arXiv:hep-ph/9409422

Chiral effective theory

- The pion mass term due to explicit symmetry breaking is included as a perturbation
- $\Longrightarrow \mathcal{L}_{eff}$ is an expansion in powers of momenta and mass of the pions (ChPT)
- Non-Goldstone fields e.g. the triplet of ρ fields transform as

$$\rho'_\mu = h\rho_\mu h^\dagger \qquad h \in SU(2)_V$$

- Interaction terms are introduced through field combinations invariant under appropriate representations of the symmetry transformations
- The lowest order interaction involving the ρ, π, ω etc

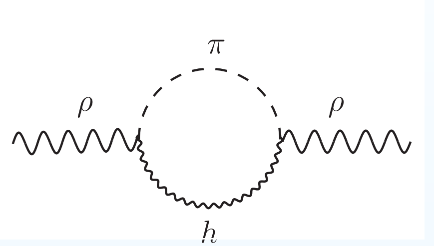
$$\begin{aligned}\mathcal{L}_{int} \quad &= \quad -\frac{2G_\rho}{m_\rho F_\pi^2}\partial_\mu\vec{\rho}_\nu \cdot \partial^\mu\vec{\pi} \times \partial^\nu\vec{\pi} \\ &\quad + \frac{g_1}{F_\pi}\epsilon_{\mu\nu\lambda\sigma}(\partial^\nu\omega^\mu\vec{\rho}^\lambda - \omega^\mu\partial^\nu\vec{\rho}^\lambda)\cdot\partial^\sigma\vec{\pi} \\ &\quad + \frac{g_2}{F_\pi}(\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu)\cdot\vec{a}_1^\mu \times \partial^\nu\vec{\pi}\end{aligned}$$

ρ self-energy (mesons)

- The one-loop self energy (in vacuum) is given by

$$\Sigma_{\mu\nu}(E,q) = i\int \frac{d^4k}{(2\pi)^4} N_{\mu\nu} D_\pi(k)\, D_h(q-k)$$

$$D(k) = \frac{1}{k^2 - m^2 + i\epsilon}$$



- To be evaluated in the medium using Thermal Field Theory

- Imaginary Time Formalism *T. Matsubara, PTP 14 (1955) 351*
  - replace propagators by $\frac{1}{\omega_n^2 + \vec{k}^2 + m^2}$ with $\omega_n = \frac{2n\pi}{\beta}$
  - replace $\int \frac{d^4k}{(2\pi)^4}$ by $\frac{1}{\beta}\sum_n \int \frac{d^3k}{(2\pi)^3}$ Matsubara sum
  - self-energy $\Sigma_{\mu\nu}$ obtained for discrete (imaginary) values of energy $\longrightarrow$ analytically continued to real continuous values

ρ self-energy

- Real Time Formalism *R.L.Kobes & G. Semenoff, NPB260 (1985) 714*
  - propagator D and self energy Σ become 2×2 matrices
  - They can be diagonalised in terms of analytic functions
  - The (diagonal) self-energy function $\overline{\Sigma}$ corresponds to the (continued) ITF result
  - can be obtained from the 11-component Σ^{11}

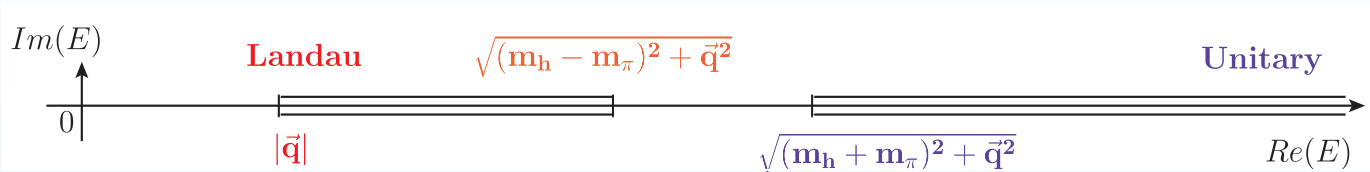
    $\mathrm{Im}\overline{\Sigma} = \tanh(\beta q_0/2)\mathrm{Im}\Sigma^{11}$

    $\mathrm{Re}\overline{\Sigma} = \mathrm{Re}\Sigma^{11}$
  - where $\Sigma^{11}_{\mu\nu}(E,q) = i\int \frac{d^4k}{(2\pi)^4} N_{\mu\nu} D^{11}_{\pi}(k)\; D^{11}_h(q-k)$
  - with $D^{11}(k) = \frac{1}{k^2 - m^2 + i\epsilon} - 2i\pi n\delta(k^2 - m^2)$

    vacuum + medium

ρ self-energy

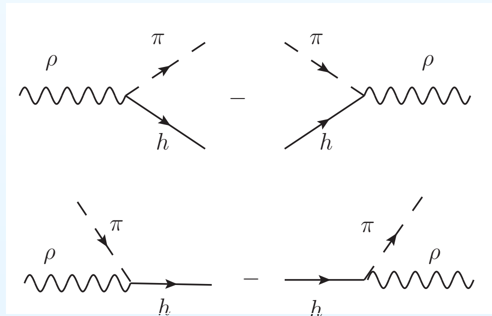
- Discontinuities in $\overline{\Sigma} \Longrightarrow$ imaginary part
- Two regions (cuts) for $E > 0$ and $q^2 > 0$



$$\mathrm{Im}\overline{\Sigma}(E,\vec{q}) = -\pi \int \frac{d^3\vec{k}}{(2\pi)^3 4\omega_\pi \omega_h} \times$$

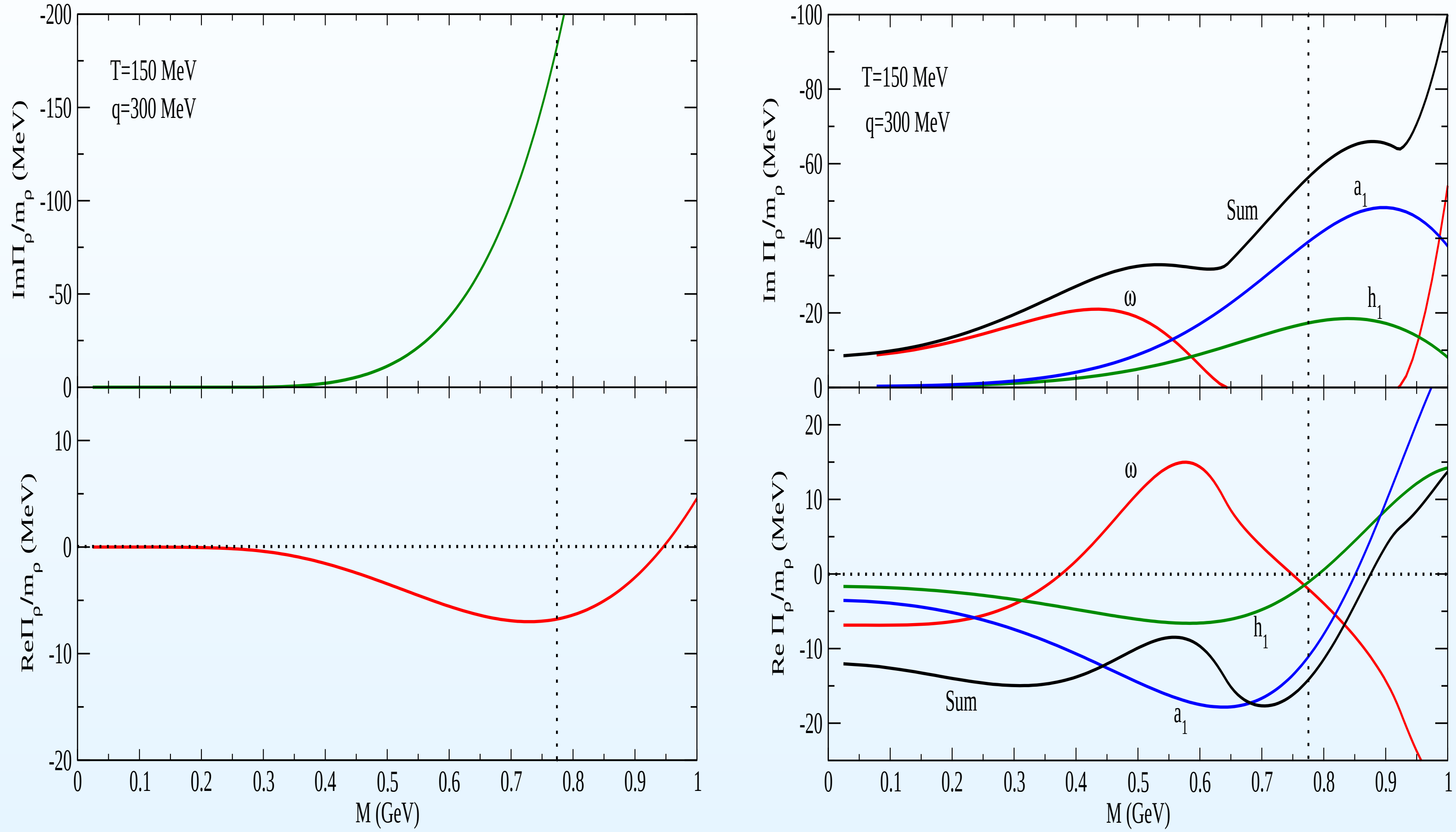
$$\Big[N_1 [(1+n_\pi)(1+n_h) - n_h n_\pi] \delta(E - \omega_\pi - \omega_h)$$

$$+ N_2 [n_\pi (1+n_h) - n_h(1+n_\pi))] \delta(E + \omega_\pi - \omega_h) \Big]$$



- δ-functions define non-zero regions $\Rightarrow$ physical processes contributing to loss or gain of ρ mesons in the medium
- Real part obtained from dispersion integral: $\mathrm{Re}\overline{\Sigma}(E,\vec{q}) = \mathcal{P} \int_0^\infty \frac{d\omega^2}{\pi} \frac{\mathrm{Im}\overline{\Sigma}(\omega,\vec{q})}{\omega^2 - E^2}$

ρ self energy



- contribution from $\pi - \pi$ loop to real and imaginary parts
- additional contributions from the $\pi - \omega$, $\pi - h_1$ and $\pi - a_1$ loops

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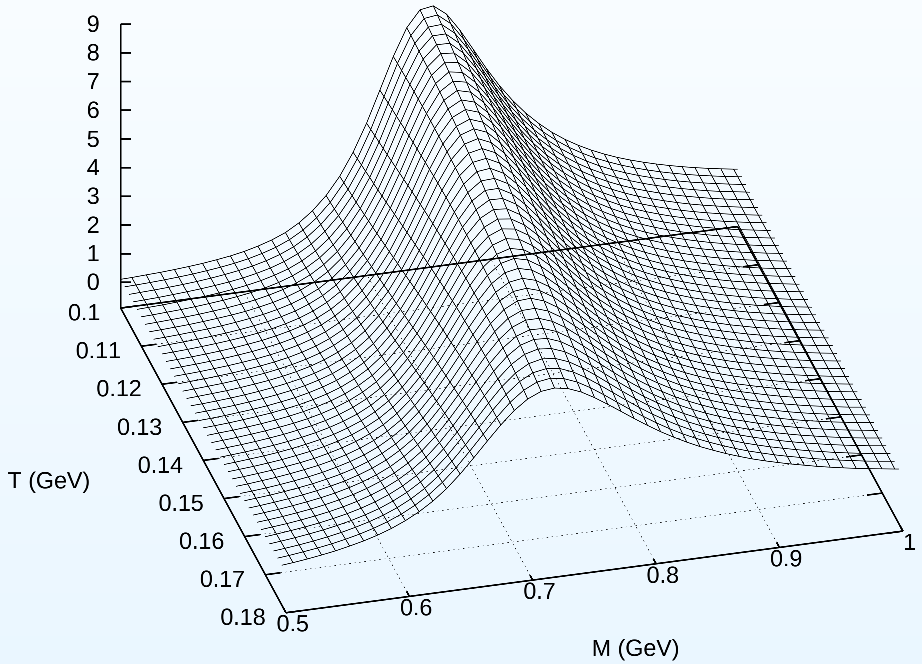
Spectral Function

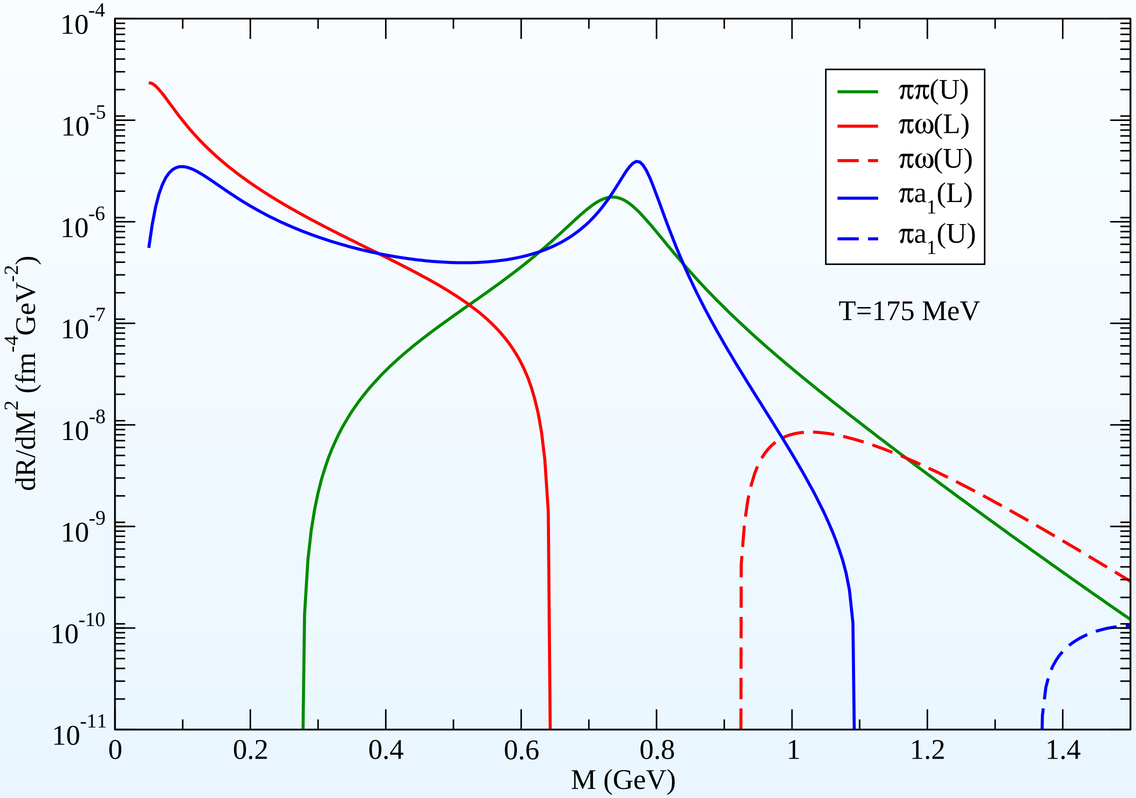
- For a hot meson gas, with $h = \pi, \omega, h_1, a_1$ mesons



- The ρ spectral function (for $|\vec{q}| = 300$ MeV) shows sizeable broadening with small mass shift

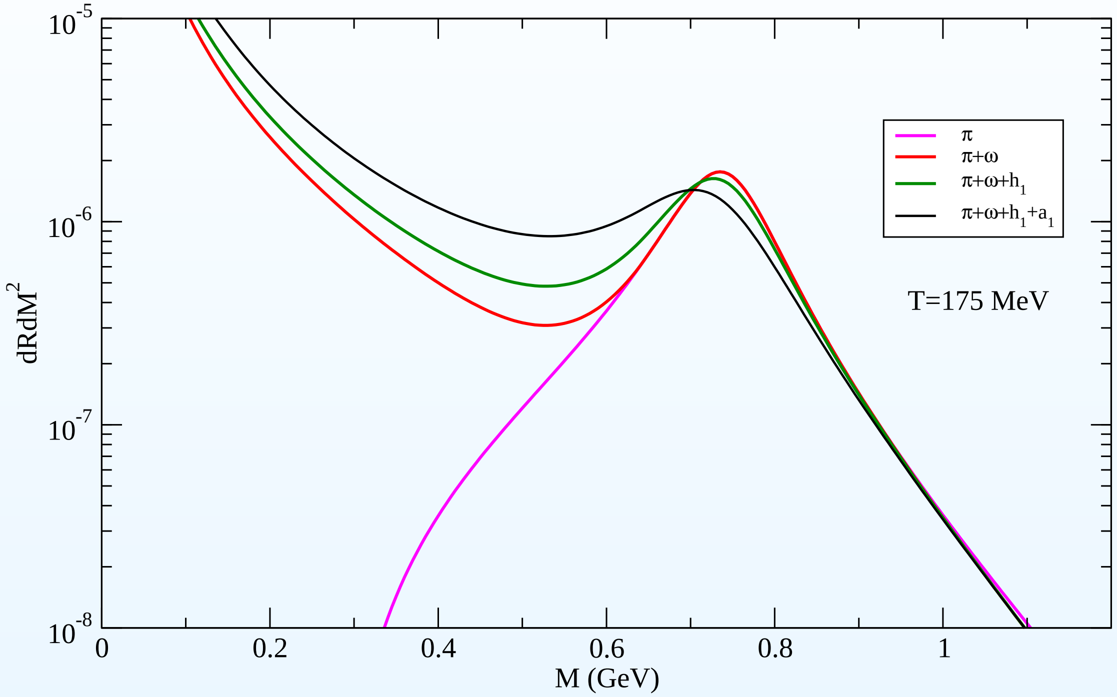
S. Ghosh et al arXiv:1009.1260

Dilepton rate (ρ only)



- The individual contributions from the Landau and unitary cuts from the $\pi - \pi$, $\pi - \omega$, $\pi - a_1$ self-energies

Dilepton rate (ρ only)

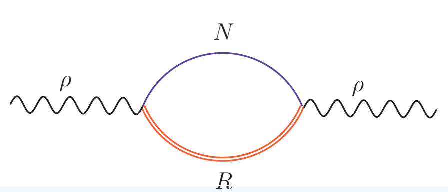


- Enhancement in the low mass dilepton rate due to spectral changes
- broadening in low mass region due to scattering processes involving heavy mesons
  $\Rightarrow$ Landau cut contributions

details in Sabyasachi's talk on 8th

Baryon Loops

- Baryon contribution is included through RN loops
- $R \equiv \Delta(1232), N^*(1520), \Delta(1650), N^*(1700)$ etc.
- The $\Delta - N - \rho$ interaction e.g.



$$\mathcal{L}_{int} = \frac{g}{F_\pi}\bar{\psi}^\mu_\Delta \gamma^\nu \psi_N \rho_{\mu\nu} \qquad J^P = \frac{3}{2}^+$$

- The relevant part comes the Landau-type discontinuity in the domain $E > 0$ and $q^2 > 0$

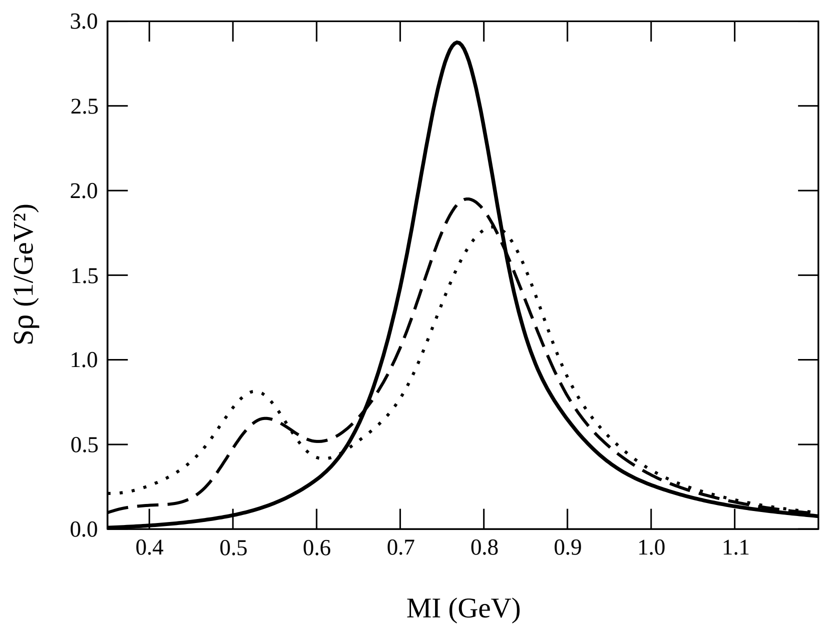
$$\mathrm{Im}\overline{\Sigma}(E,\vec{q}) = -\pi \int \frac{d^3\vec{k}}{(2\pi)^3 4\omega_N\omega_R}\left[(N_1 n_+^R + N_2 n_-^R) - (N_3 n_+^N + N_4 n_-^N)\right]\delta(E+\omega_N-\omega_R)$$

where $n_+ = \dfrac{1}{e^{\beta(E-\mu)}+1} \rightarrow$ baryons

and $\quad n_- = \dfrac{1}{e^{\beta(E+\mu)}+1} \rightarrow$ anti-baryons

- Contributes even at $\rho_N = 0$ because contributions from baryons and anti-baryons appear additively

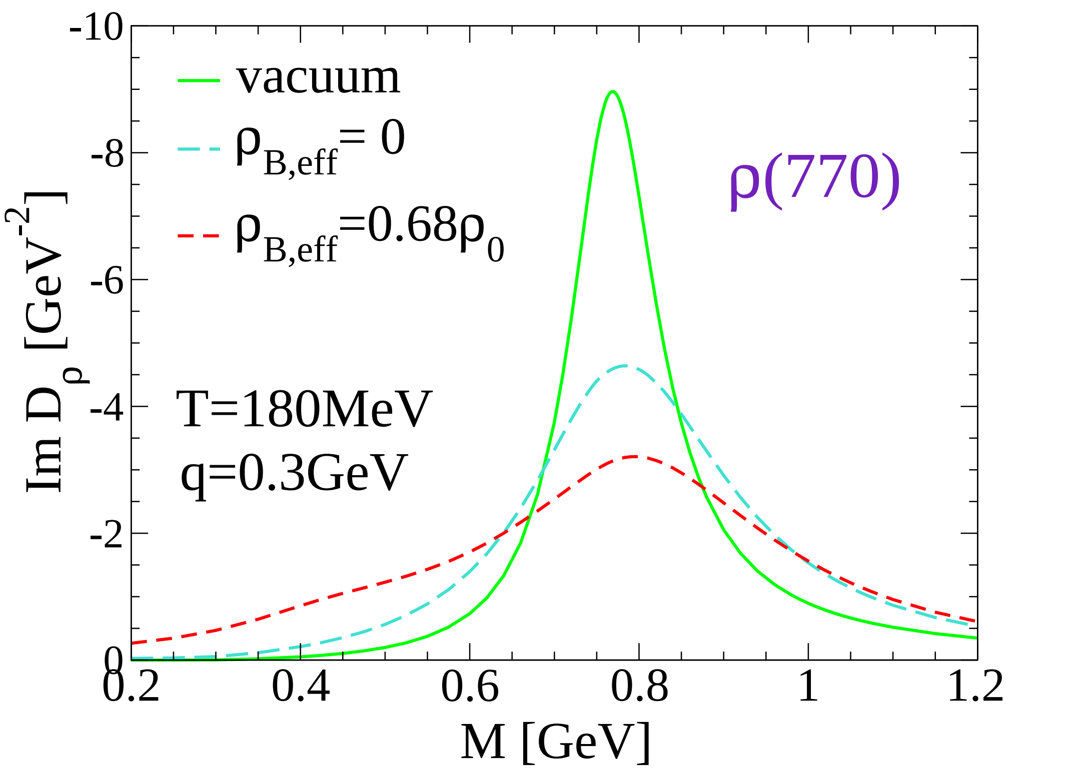
ρ spectral function in dense matter at $T = 0$



D. Cabrera et al NPA 705 (2002) 90

- rho spectral function in dense matter at $\rho = 0$, $\rho = \rho_0/2$ and $\rho = \rho_0$ in a chiral approach involving $\Delta(1230)$ and $N^*(1520)$
- New structure at low mass from Landau-type discontinuities in the $N^*(1520) - N$ self-energy

ρ spectral fn in hot & dense matter



R. Rapp et al arXiv:0901.3289

- rho spectral function in hot and dense matter calculated in a many-body approach involving mesons and baryons
- substantial broadening $\Rightarrow$ melting of ρ

Dilepton spectra

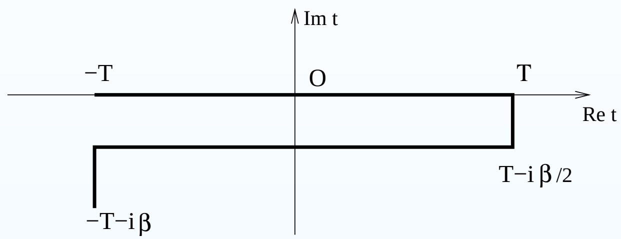
- More work needs to be done to obtain the low mass dilepton yield to be compared with experimental data
  - In addition to the ρ, the in-medium spectral functions of the ω and possibly ϕ are required for the rate of emission from hadronic matter
  - rate of emission from QGP
  - convolution over the space-time history of the fireball using relativistic hydrodynamics
  - a realistic equation of state
  - implementation of chemical and kinetic freeze-out
  - fold over the Acceptance function of the detector, if any

More on this in Jan-e Alam's talk on 9th

Concluding remarks

- The study of hadrons in medium provides a handle to study non-perturbative phenomena like chiral phase transition in QCD
- However, we should keep in mind that not every in-medium change in the properties of hadrons is related to chiral symmetry restoration
- change due to purely hadronic many body effects like scattering and decay in the medium
- It is not sensible to try to determine what part of the medium effect has a 'conventional' origin and how much is related to chiral symmetry breaking/restoration
- Essential to carefully and exhaustively evaluate the in-medium correlation functions with chiral effective interactions in a Quantum Field Theoretic framework
- This needs to be corroborated with LQCD simulations as well as constraints coming from the sum rules

Real Time Formalism



- The free propagator

$$D^{11} = -(D^{22})^* = \Delta(k_0, \vec{k}) + 2\pi i n \delta(k^2 - m^2)$$
$$D^{12} = D^{21} = 2\pi i \sqrt{n(1+n)}\delta(k^2 - m^2)$$

where $\Delta(k_0, \vec{k}) = \dfrac{-1}{k^2 - m^2 + i\epsilon}$

- The thermal propagator may be diagonalised in the form

$$D^{ab}(k_0, \vec{k}) = U^{ac}(k_0)[\mathrm{diag}\{\Delta(k_0, \vec{k}), -\Delta^*(k_0, \vec{k})\}]^{cd} U^{db}(k_0)$$

with the elements of the diagonalising matrix as

$$U^{11} = U^{22} = \sqrt{1+n}, \quad U^{12} = U^{21} = \sqrt{n}$$

Real Time Formalism

- From spectral representations, one can show that U diagonalises also the full propagator

- As a consequence, the matrix Σ^{ab} is also diagonalisable by $(U^{-1})^{ab}$,

$$\Sigma^{ab}(q) = [U^{-1}(q_0)]^{ac}[\mathrm{diag}\{\overline{\Sigma}(q), -\overline{\Sigma}^*(q)\}]^{cd}[U^{-1}(q_0)]^{db}$$

- The diagonal component can be obtained from the 11-component Σ^{11} as $\mathrm{Im}\overline{\Sigma} = \tanh(\beta q_0/2)\mathrm{Im}\Sigma^{11}$

  $\mathrm{Re}\overline{\Sigma} = \mathrm{Re}\Sigma^{11}$

- The diagonal components (barred quantities) satisfy the same Dyson equation as the matrix form

$$\overline{D} = \overline{D}_0 + \overline{D}_0\ \overline{\Sigma}\ \overline{D}$$