The QCD phase diagram from the lattice

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ILGTI: TIFR

ICPAGQP Student Day
Doan Paula, Goa
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Outline

How many flavours

- Flavour symmetries not exact: difference in masses of different flavours breaks symmetry. For example: if $m_u = m_d$ then (within QCD) any two mutually orthogonal linear combinations of u and d are equivalent: symmetry. When $m_u \neq m_d$, symmetry broken: these transformations change energy.
- Since $m_{\pi^0} \simeq m_{\pi^\pm}$, flavour SU(2) is a good approximate symmetry of the hadron world. Flavour SU(3) is not useful without symmetry breaking terms Gell-Mann and Nishijima
- If some $m \gg \Lambda_{QCD}$ then that quark is not approximately chiral. In QCD two flavours are light ($m_{u,d} \ll \Lambda_{QCD}$) and one is medium heavy ($m_s \simeq \Lambda_{QCD}$). Recall that $m_\pi = 0.2 m_\rho$ but $m_K = 0.7 m_\rho$. Limit $m_\pi = 0$: chiral symmetry.
- Do we have a two flavour phase diagram or a three flavour phase diagram, or something else?

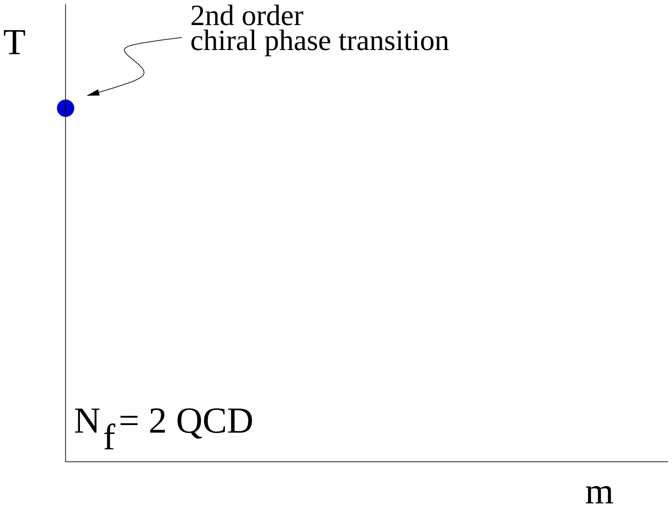
The two flavour world

- What distinguishes the phases? In the vacuum chiral symmetry is broken; $\langle\overline{\psi}\psi\rangle$ **chiral condensate** is non-vanishing. At high temperature $\langle\overline{\psi}\psi\rangle = 0$ if quarks are massless. Critical only in this case.
- When correlation lengths finite then susceptibility always finite:

  $$\chi = \int d^3x C(x), \quad C(x) \simeq \exp(-mx), \quad \xi = 1/m_{PS}.$$

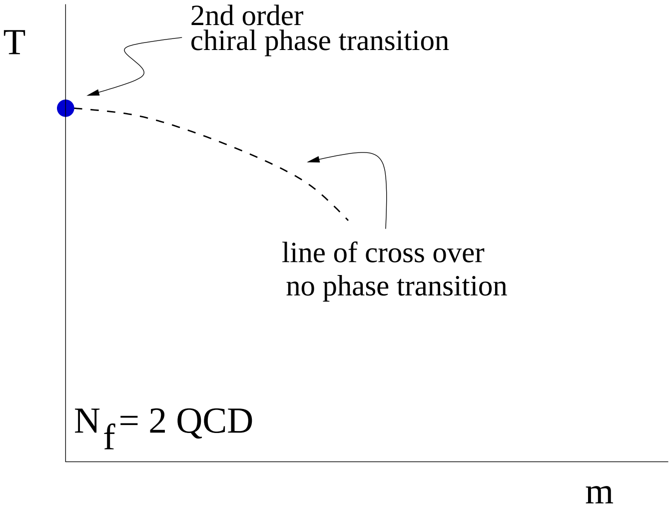
  At critical points correlation lengths diverge; integral diverges, so susceptibility diverges.
- When quarks are massive, then at transition scalar mass degenerate with $m_\pi \neq 0$. No vanishing masses, so all susceptibilities finite. May still have a maximum as T changes: cross over for massive quarks.

The two flavour phase diagram



Pisarski and Wilczek, PR D 29, 338 (1984)

The two flavour phase diagram



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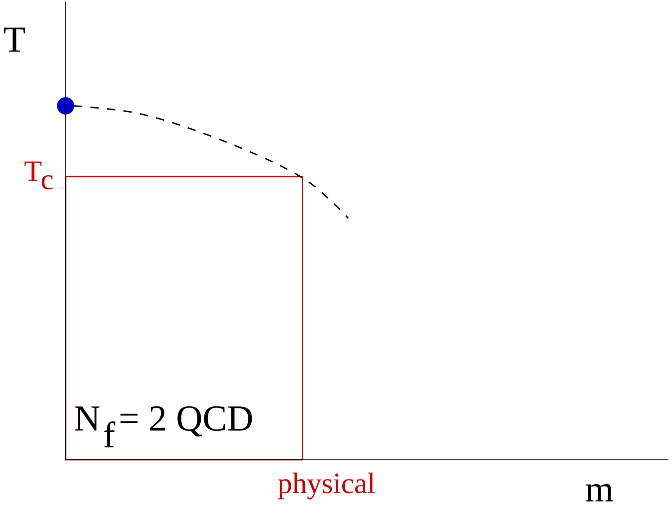

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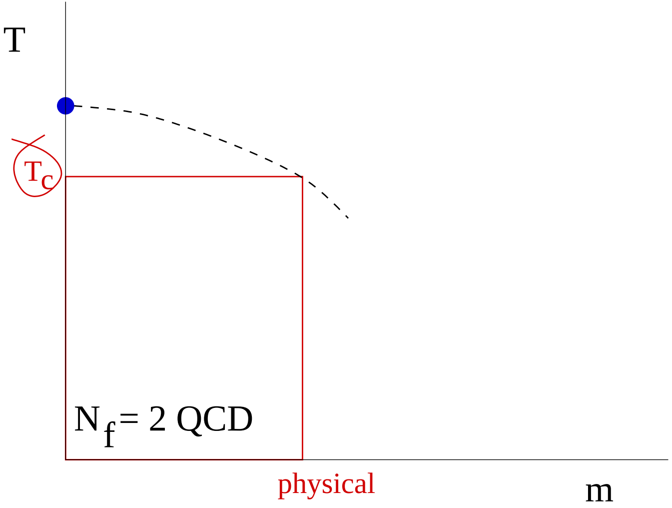

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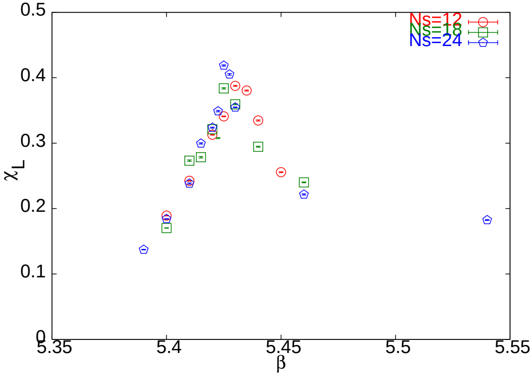
Cross over and deconfinement transition



Wilson line susceptibility measures T_c Gavai, SG: 2008

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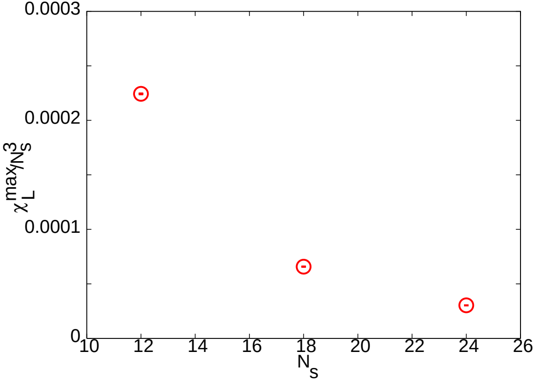
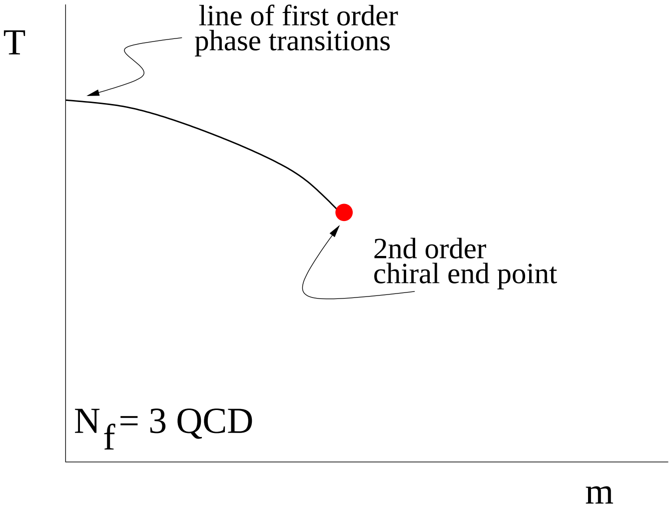


Wilson line susceptibility measures T_c Gavai, SG: 2008

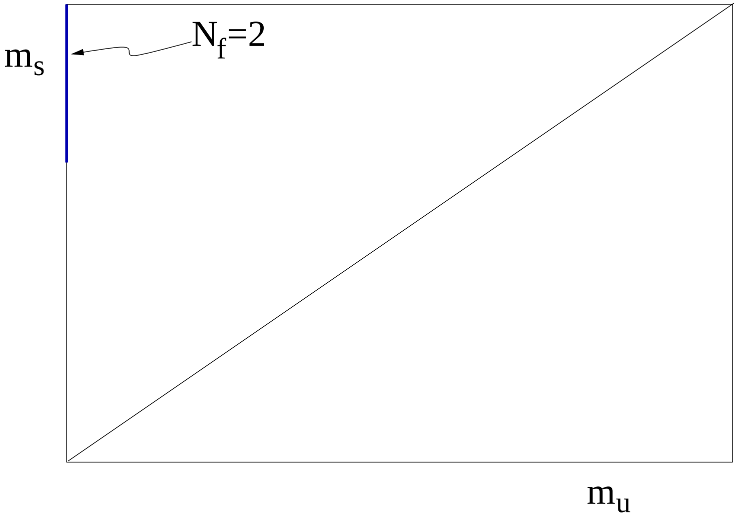
The three flavour world

- For three massless flavours $m_\pi = m_K = m_\eta = 0$. Chiral condensate distinguishes phases. However: first order phase transition; chiral condensate vanishes with a jump.
- If flavour symmetry exact ($m_\pi = m_K = m_\eta \neq 0$), then first order transition remains stable upto some point. Jump decreases continuously until it vanishes. This is a critical end point of this line.
- In the real world SU(3) flavour symmetry is broken ($m_\pi \neq m_K \neq m_\eta$). What is the phase diagram? Encoded in the Columbia plot.

The three (degenerate) flavour phase diagram



The Columbia plot



Brown et al, PRL 65, 2491 (1990)

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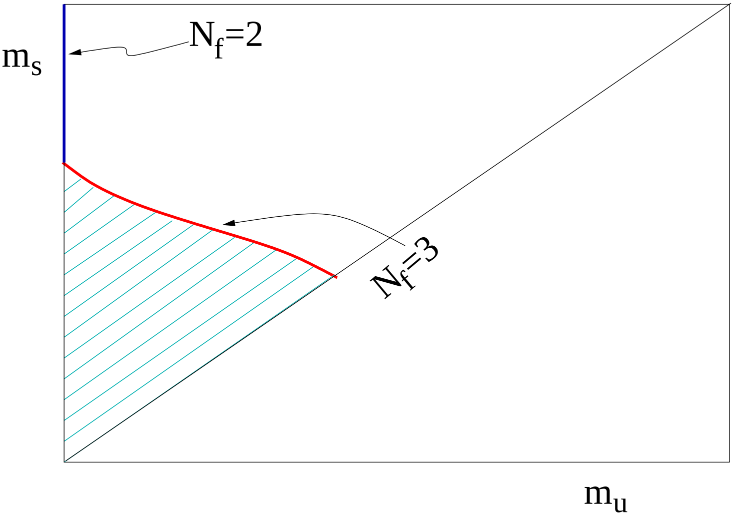

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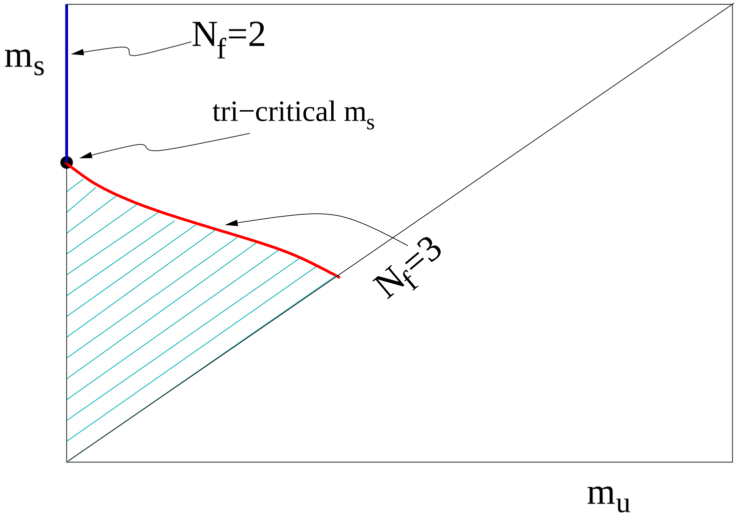

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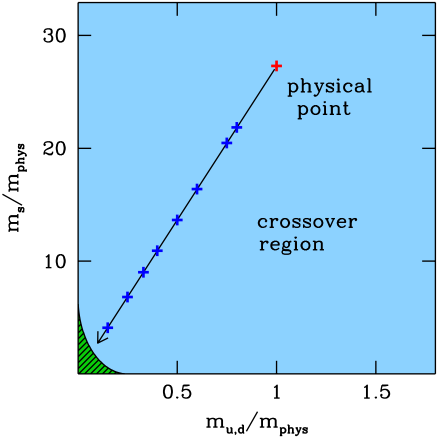
Lattice results for the Columbia Plot



In $N_f = 2 + 1$:

$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988 (2007)

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121 (2004)

Lattice results for $N_f = 2 + 1$

1. Two independent lattice computations (now) agree on the position of the crossover temperature for physical quark mass ($m_\pi \simeq 140$ MeV):
$$T_c \simeq 170 \text{ MeV}.$$
Aoki etal, hep-lat/0611014 (2006); HotQCD, 2010.
2. Clear evidence that susceptibilities do not diverge: no critical point, definitely a cross over. BW: difference between "chiral" and "deconfinement" cross overs. HotQCD: no such difference.
3. Chiral ($m_\pi = 0$) critical point: not yet well determined. Current studies indicate that expected divergences do occur. However, the approach to infinities are not yet completely under control. Ejiri etal, 0909.5122 (2009)

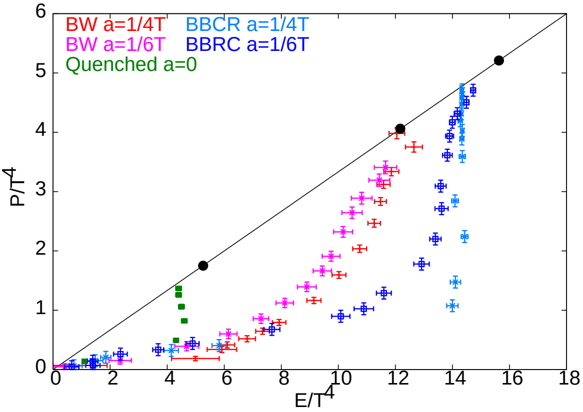
Lattice results for $N_f = 1 + 1$

No significant change in T_c as $m_{\pi^0}/m_{\pi^\pm}$ is changed from 1 to 0.78 (physical value bracketed). Only one study; lattice spacings are coarse by today's standards; finite size scaling yet to be performed.
Gavai, SG, hep-lat/0208019 (2002)

$$\frac{T_c}{\Lambda_{\overline{MS}}} = \begin{cases} 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 1) \\ 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 0.78) \end{cases}$$

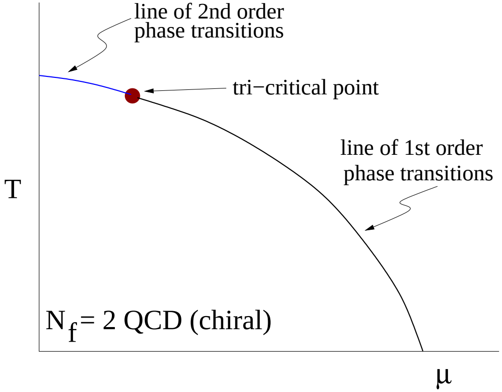
Both results extrapolated to the physical value of m_π/m_ρ.

The equation of state



Outline

The two flavour phase diagram



Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

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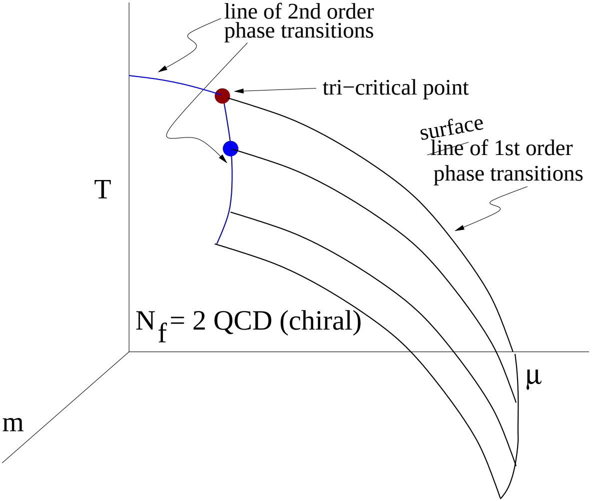

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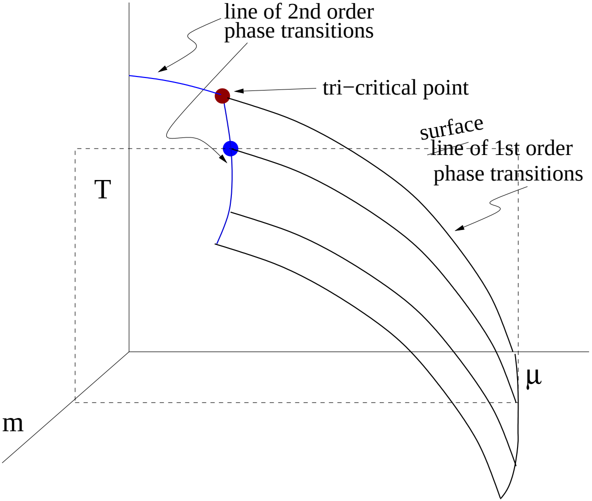

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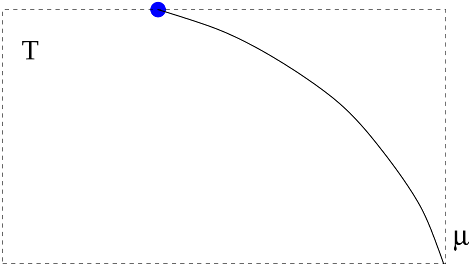

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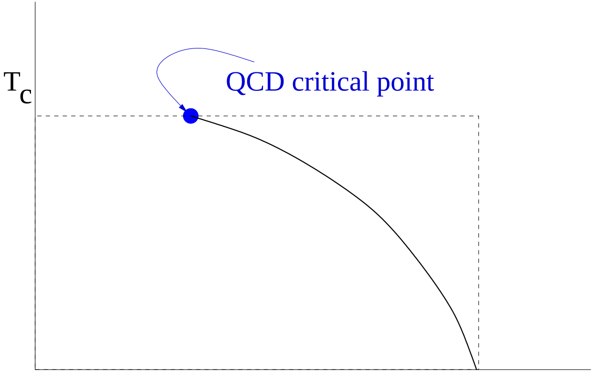


Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

Lattice setup

Lattice simulations impossible at finite baryon density: **sign problem**. Basic algorithmic problem in all Monte Carlo simulations: no solution yet.

Bypass the problem; make a Taylor expansion of the pressure:

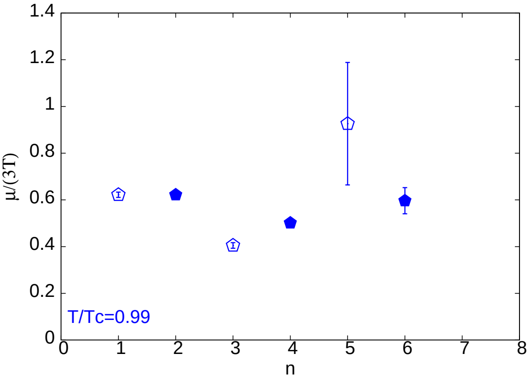
$$P(T,\mu) = P(T) + \chi_B^{(2)}(T)\frac{\mu^2}{2!} + \chi_B^{(4)}(T)\frac{\mu^4}{4!} + \cdots$$

Series expansion coefficients evaluated at $\mu = 0$.
Implies

$$\chi_B^2(T,\mu) = \chi_B^{(2)}(T) + \chi_B^{(4)}(T)\frac{\mu^2}{2!} + \chi_B^{(6)}(T)\frac{\mu^4}{4!} + \cdots$$

Series fails to converge at the critical point.
Gavai, SG, hep-lat/0303013 (2003)

Series can diverge



Radius of convergence of the series as a function of order ($a^{-1} = 1200$ MeV)

Gavai, SG, 0806.2233 (2008)

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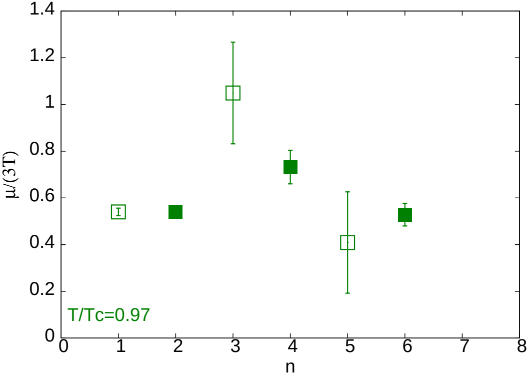

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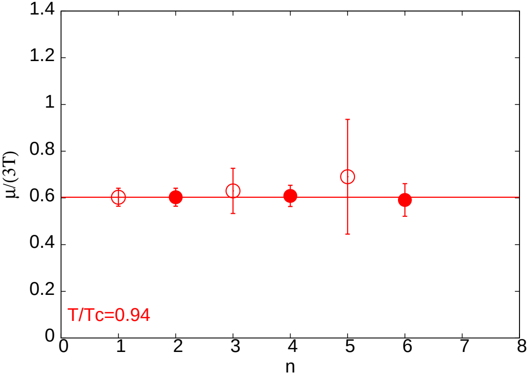

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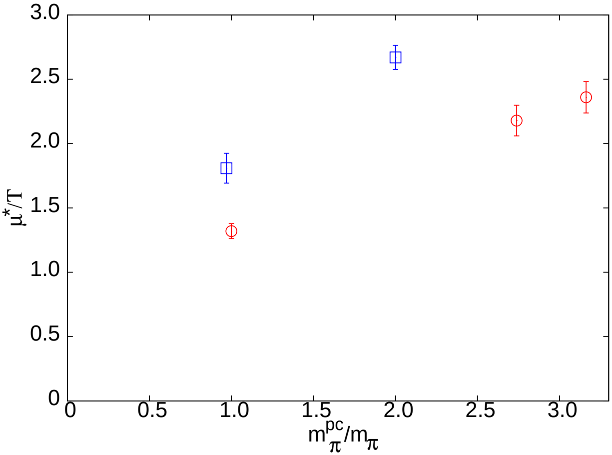
Dependence on quark mass



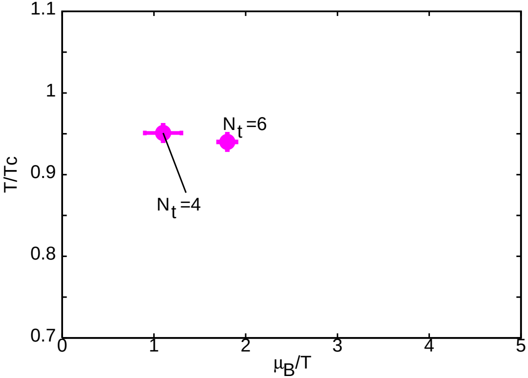
$a^{-1} = 800$ MeV, 1200 MeV

SG, hep-lat/0608022 (2006) and unpublished

Systematic effects

1. Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects. Finite size scaling tested; works well
   Gavai, SG 2004, 2008
2. What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks
   Gavai, SG, hep-lat/0510044; see also RBRC 2009; de Forcrand, Philipsen, 2007, 2009
3. What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in
   Gavai, SG, Ray, nucl-th/0312010; see also Fodor, Katz 2001, 2002.
4. What happens in the continuum limit? Estimate of μ_B^E may increase somewhat
   Gavai, SG 2008; SG 2009

The critical point of QCD



$$\frac{\mu^E}{T^E} \simeq \begin{cases} 1.8 \pm 0.1 & N_f = 2,\ 1/a = 1200 \text{ MeV} \text{ Gavai, SG, 0806.2233 (2008)} \\ 1.5 \pm 0.4 & N_f = 2+1,\ 1/a = 800 \text{ MeV} \text{ Schmidt, 2010} \end{cases}$$

comparable m_π; normalized to same estimator.

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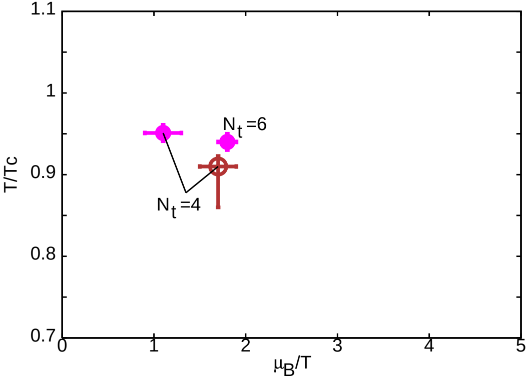

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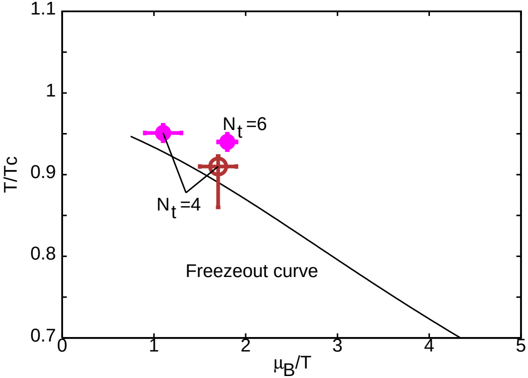

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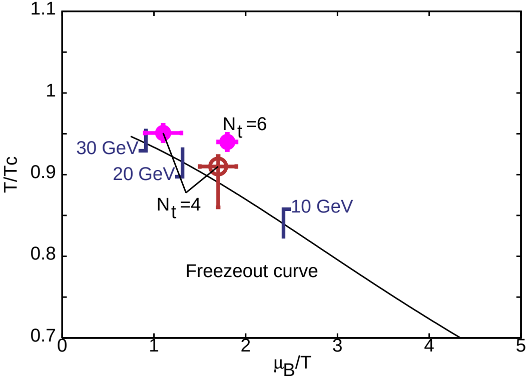

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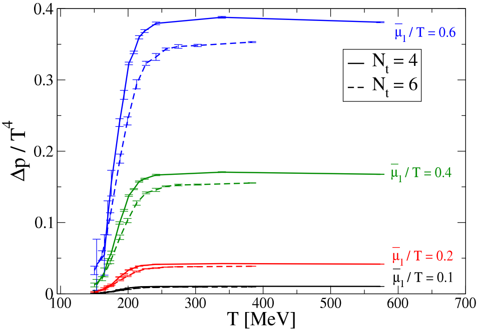
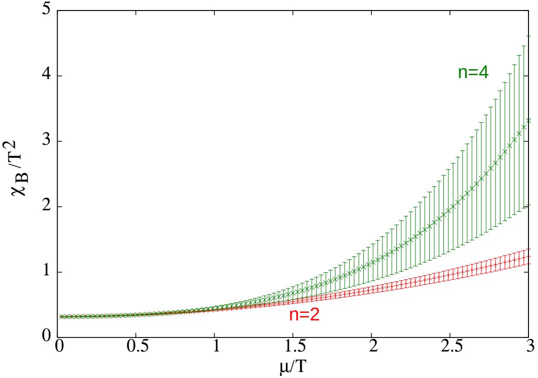
Extrapolating physical quantities



$$\Delta p = \chi_B^{(2)} \frac{\mu^2}{2!} + \chi_B^{(4)} \frac{\mu^4}{4!} + \cdots - \chi_S^{(2)} \frac{\mu_S^2}{2!} - \cdots$$

MILC Collaboration, 1003.5682 (2010)

Critical divergence: summation bad, resummation good



Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful Gavai, SG, 0806.2233 (2008).

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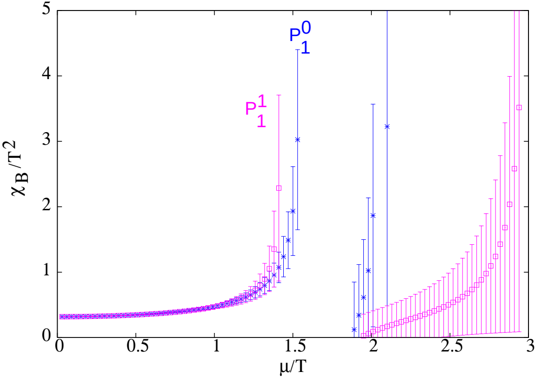

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Outline

Experimentalists know how to make fireballs

" We didn't have flint when when I was a kid, we had to rub two sticks together. "

Locating the critical end point in experiment

Measure the (divergent) width of momentum distributions
Stephanov, Rajagopal, Shuryak, hep-ph/9903292 (1999)
Better idea, use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B \rangle.$$

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution:
central limit theorem
Landau and Lifschitz
Bias-free measurement possible
Asakawa, Heinz, Muller, hep-ph/0003169 (2000); Jeon, Koch, hep-ph/0003168 (2000).

Is the top RHIC energy non-critical?

Check whether CLT holds.

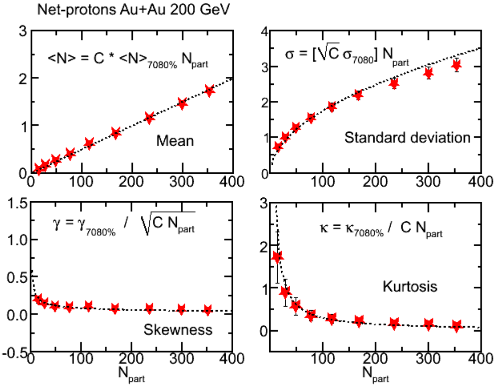
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Coefficients depend on T and μ. So make sure that the nature of the physical system does not change while changing the volume.

This is a check that microscopic physics is forgotten (except two particle correlations).

STAR measurements



Perfect CLT scaling: remember only $VT\chi_B$? or some other physics?

Can we recover microscopic physics?

Can we test QCD?

STAR Collaboration: QM 2009, Knoxville

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3 V \chi^{(4)}.$$

T and V are unknown, so direct measurement of QNS not possible (yet). Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD provided all other fluctuations removed.

SG, 0909.4630 (2009)

Tests and assumptions

$$m_1: \quad \frac{[B^3]}{[B^2]} = \frac{\chi^{(3)}(T,\mu_B)/T}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_2: \quad \frac{[B^4]}{[B^2]} = \frac{\chi^{(4)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_3: \quad \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)/T}$$

Also for cumulants of electric charge, Q, and strangeness, S.

1. Two sides of the equation equal if there is thermal equilibrium and no other sources of fluctuations.
2. Right hand side computed in the grand canonical ensemble (GCE). Can observations simulate a grand canonical ensemble? What T and μ_B?
3. Why should hydrodynamics and diffusion be neglected?

Why thermodynamics and not dynamics?

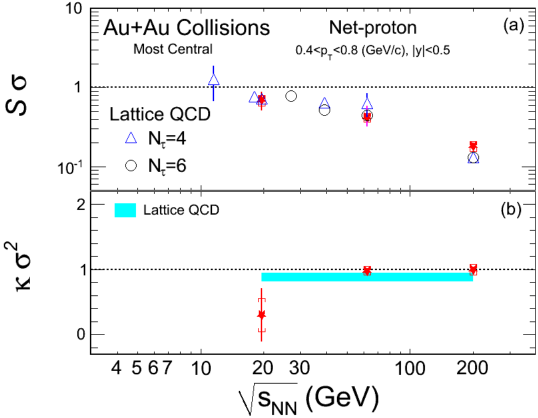
Chemical species may diffuse on the expanding background of the fireball, so why should we neglect diffusion and expansion?
Bhalerao, SG: 2009

First check whether the system size, ℓ, is large enough compared to the correlation length ξ: Knudsen's number $K = \xi/\ell$. If $K \ll 1$, ie, $\ell \gg \xi$ then central limit theorem will apply.
Next, compare the relative importance of diffusion and advection through a dimensionless number (Peclet's number):

$$\mathcal{W} = \frac{\ell^2}{t\mathcal{D}} = \frac{\ell v_{flow}}{\mathcal{D}} = \frac{\xi v_{flow}}{K\mathcal{D}} = \frac{v_{flow}}{Kc_s} = \frac{M}{K}.$$

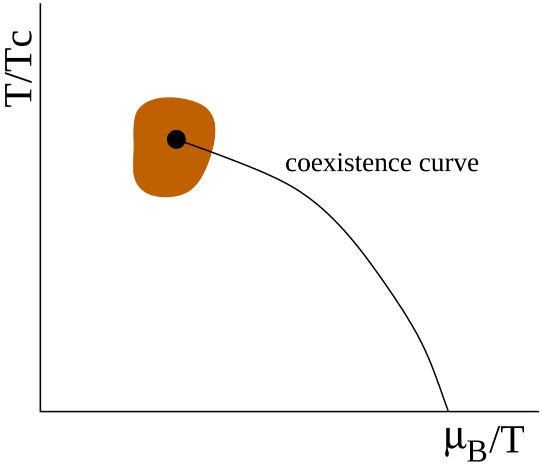
When $\mathcal{W} \ll 1$ diffusion dominates. After chemical freeze-out K is small but Mach's number $M \simeq 1$, so flow dominates: fluctuations are frozen in. So detector observes thermodynamic fluctuations at chemical freeze out.

STAR tests non-perturbative QCD



STAR Collaboration, 1004.4959 (2010)

How to find the critical point: be lucky!



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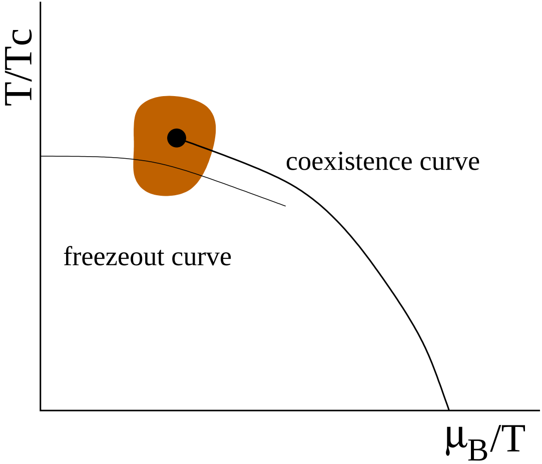

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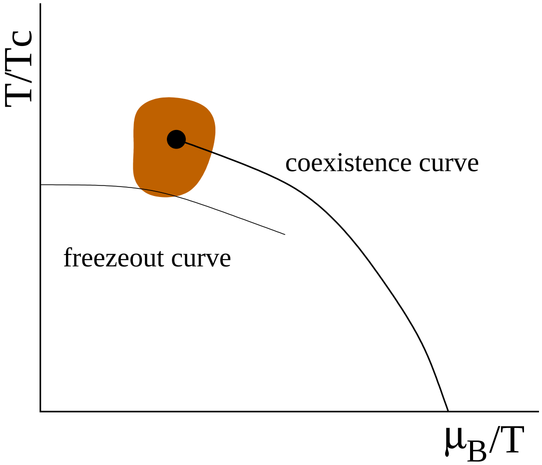

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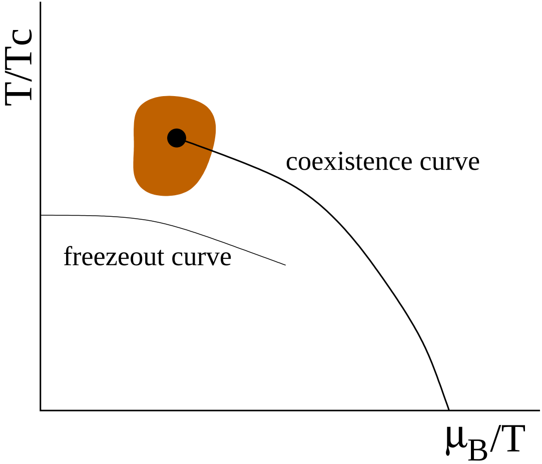
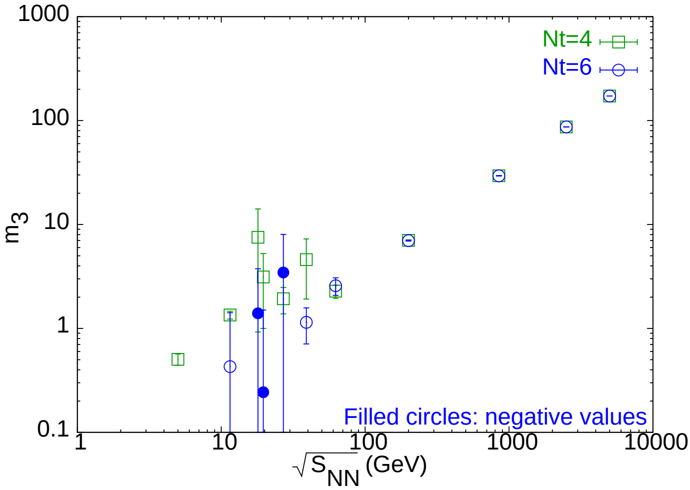


Lattice predictions along the freezeout curve



Gavai, SG, 1001.3796 (2010)

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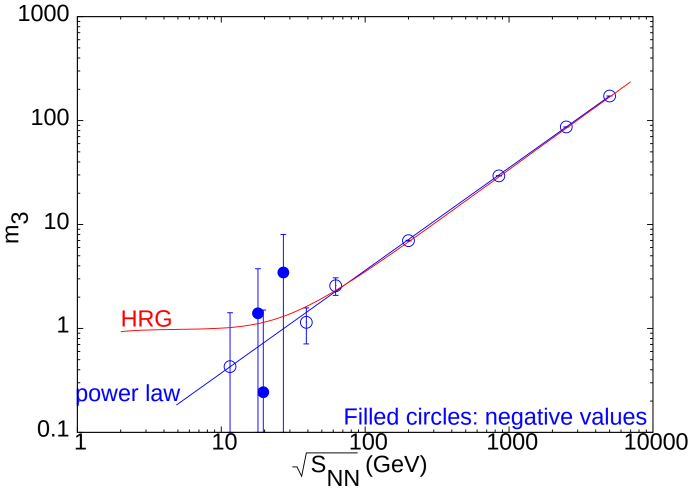

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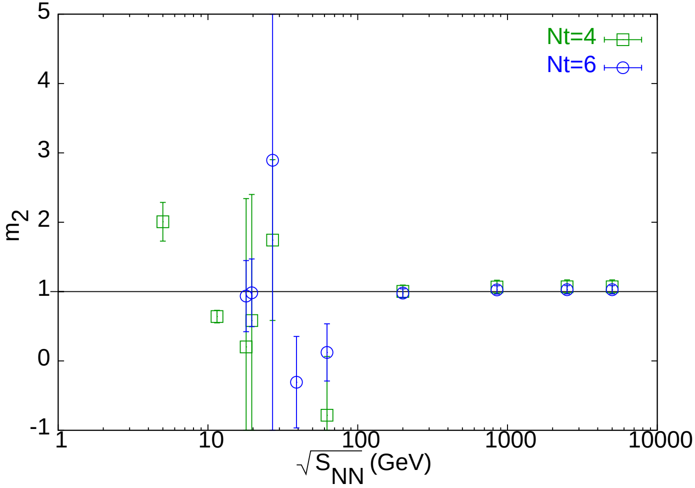

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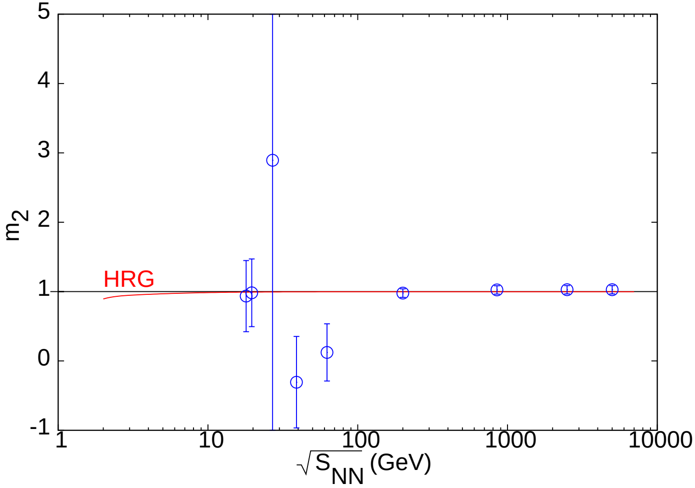

Gavai, SG, 1001.3796 (2010)

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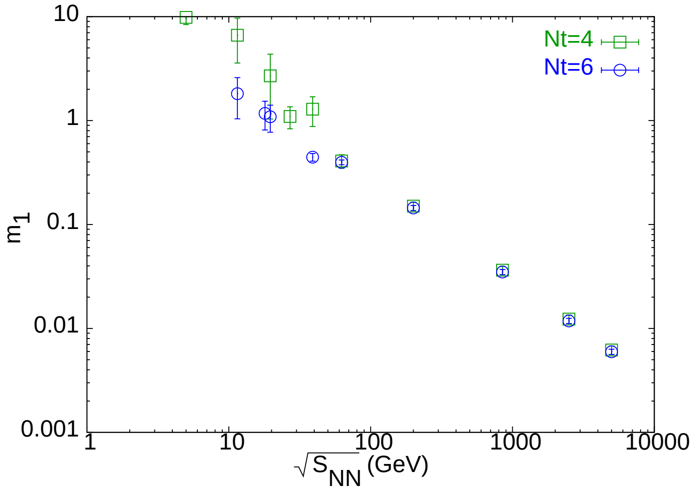

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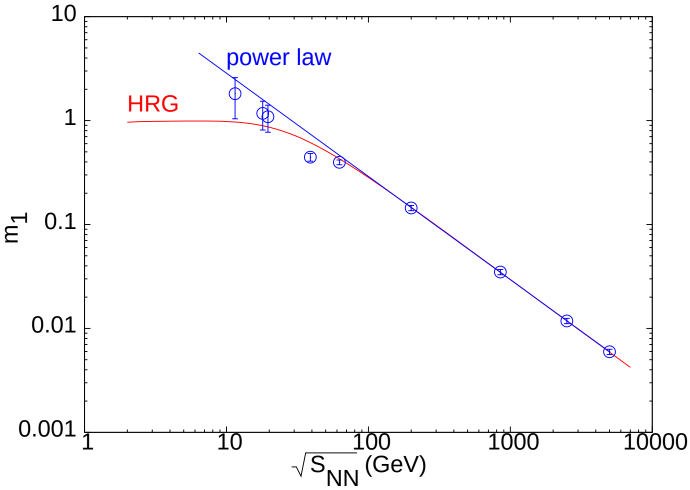

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Finite size effects damp divergences

1. System size limits correlation lengths near the critical point: $\ell \simeq \xi$. The Knudsen number is never small near the CEP, so central limit theorem will stop working. Check the scaling of σ^2, $\mathcal{S}$ and $\mathcal{K}$ and see whether there are violations of the central limit theorem. (SG: 2009)
2. As a result, the Peclet number need not be large, and diffusion may play an important role even close to kinetic freeze-out. Then fluctuations of conserved quantities may not be comparable to thermal equilibrium values at chemical freeze-out!
3. Another way of saying this is: critical divergences are limited due to finite size effects: no singularities, hence no direct measurement of the critical exponents. System drops out of equilibrium due to finite lifetime. (Stephanov: 2008; Berdnikov, Rajagopal: 1998)

Outline

Main results

- The strange quark is heavy; light quarks determine the shape of the phase diagram. The cross over temperature now under control: $T_c \simeq 170$ MeV. SU(2) flavour symmetry breaking unlikely to change T_c.
- Lattice determines series expansion of pressure; indicates a critical point in QCD. Lattice spacing effects under reasonable control. Physical quantities can be found be resumming the series expansion (*e.g.*, Padé approximants).
- First direct comparison of lattice results with experimental data done; good agreement. A landmark in the field: good evidence for thermalization.
- A step-by-step analysis suggested for critical point: failure of CLT scaling, fluctuations not frozen at chemical freezeout, evidence for non-monotonic behaviour of $m_{1,2,3}$ near this point.