Nuclear matter and neutron star matter with quark matter phase transition in the effective chiral model.

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Introduction

- RMF is very successful at around the saturation density and in finite nuclei within the range of the model parameters.
- There are uncertainties in the stiffness and the high density behavior of the equation of state (EOS).
- Thus, it is very important to develop relativistic models with constraints to this uncertainty by some symmetries, and to investigate bulk properties of nuclear systems.
- *One of the most important symmetries in hadron physics is the chiral symmetry. Chiral symmetry is a fundamental symmetry in QCD with massless quarks, and its spontaneous breaking generates hadron masses through the chiral condensate* $\langle q\bar{q}\rangle$ *which is considered to be partially restored in dense matter.*

- When we naïvely introduce the vector meson field into the ϕ^4 theory, however, it is known that the normal vacuum jumps to a chiral restored abnormal vacuum (Lee-Wick vacuum) below the saturation density, and this problem is referred to as the chiral collapse problem.

- One of the prescriptions to avoid this problem is to incorporate a logarithmic term of σ in the chiral potential (energy density as a function of σ at zero baryon density).

- Another way to avoid the chiral collapse is to introduce a dynamical generation of the isoscalar-vector meson mass through the coupling between scalar and vector mesons. Since the vector meson becomes light when the chiral symmetry is partially restored, repulsive effects from the vector meson become strong and we can avoid the chiral collapse.

- One of the problems in this theoretical treatment is the unrealistically high incompressibility value,
- In order to make moderate value of incompressibility, we introduced the higher order terms of scalar meson, σ^6 and σ^8. In this way, we can reproduce the empirical values of the saturation density, binding energy, and incompressibility in symmetric nuclear matter.
- In all these works, the vacuum stability at large σ values was not critically examined for all sets of parameters.

Formalism of SU(2) Effective Chiral Model

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}\left[i\partial\!\!\!/ - g_\sigma(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_\omega \omega\!\!\!/ - g_\rho \boldsymbol{\rho}\!\!\!/\cdot\boldsymbol{\tau}\right]\psi \\
&+\frac{1}{2}\big(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \partial_\mu\sigma\partial^\mu\sigma\big) - V_\sigma(\sigma,\boldsymbol{\pi}) \\
&-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}g_{\sigma\omega}^2 x^2\omega_\mu\omega^\mu \\
&-\frac{1}{4}\mathbf{G}_{\mu\nu}\cdot\mathbf{G}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu \ .
\end{aligned}
\tag{1}
$$

We introduce a chiral symmetric type interaction up to eighth order of the meson field which reads,

$$V_\sigma = \frac{C_4 f_\pi^4}{4}\left(\frac{x^2}{f_\pi^2}-1\right)^2 + \frac{m_\pi^2}{2}x^2 - m_\pi^2 f_\pi \sigma + \frac{C_6 f_\pi^4}{6}\left(\frac{x^2}{f_\pi^2}-1\right)^3 + \frac{C_8 f_\pi^4}{8}\left(\frac{x^2}{f_\pi^2}-1\right)^4 , \quad (2)$$

The masses of the nucleon and vector meson in vacuum are given by,

$$M_N = g_\sigma f_\pi \ , \ \ m_\omega = g_{\sigma\omega} f_\pi \ , \quad (3)$$

where the vacuum expectation value of the σ field is replaced with f_π. The coefficient C_4 is related to the vacuum mass of σ as

$$C_4 = \frac{m_\sigma^2 - m_\pi^2}{2f_\pi^2} \ . \tag{4}$$

The constant parameters C_6 and C_8 are included in the higher-order self-interaction of the scalar field to describe the desirable values of nuclear matter properties at saturation point. Using the mean-field ansatz in uniform matter, the equations of motion for the vector fields (ω and ρ-mesons) are solved as,

$$\omega = \frac{g_\omega^2 \rho_B^2}{g_{\sigma\omega}^2 x^2} = \frac{g_\omega^2 \rho_B^2}{m_\omega^2 Y^2} \ , \quad R \equiv \rho_0^3 = \frac{g_\rho}{m_\rho^2}(\rho_p - \rho_n) \ , \tag{5}$$

The total energy density (ε) and pressure (P) of the uniform many-nucleon system are given by,

$$\varepsilon = \varepsilon_N(M_N^\star) + V_\sigma + \frac{g_\omega^2 \rho_B^2}{2m_\omega^2 Y^2} + \frac{1}{2} m_\rho^2 R^2 \ , \quad (6)$$

$$P = P_N(M_N^\star) - V_\sigma + \frac{g_\omega^2 \rho_B^2}{2m_\omega^2 Y^2} + \frac{1}{2} m_\rho^2 R^2 \ , \quad (7)$$

$$\varepsilon_N = \sum_{\alpha=p,n} \frac{\gamma}{2\pi^2} \int_0^{k_F^{(\alpha)}} k^2 dk \sqrt{k^2 + M_N^{\star 2}} \ , \quad (8)$$

$$P_N = \sum_{\alpha} \frac{\gamma}{6\pi^2} \int_0^{k_F^{(\alpha)}} \frac{k^4 dk}{\sqrt{k^2 + M_N^{\star 2}}} \ , \quad (9)$$

Parameters

The relation with their parameters with the present parameters are given by,

$$C_4 = \frac{M_N^2}{2c_\sigma f_\pi^4}\ ,\ C_6 = \frac{B}{2f_\pi^4 c_\sigma c_\omega}\ ,\ C_8 = \frac{C}{2f_\pi^4 c_\sigma c_\omega^2 M_N^2}\ , \qquad (10)$$

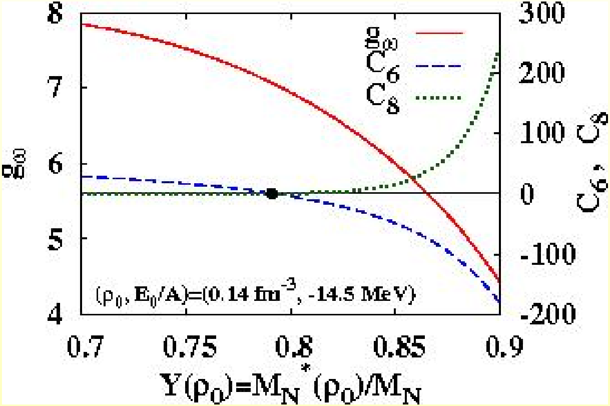
and $g_\omega = \sqrt{c_\omega} m_\omega$.

Results and Discussions

Here, we use constants $M_N = 938$ MeV, $f_\pi = 93$ MeV, $m_\omega = 783$ MeV, $m_\rho = 770$ MeV and $g_\sigma = M_N/f_\pi$.

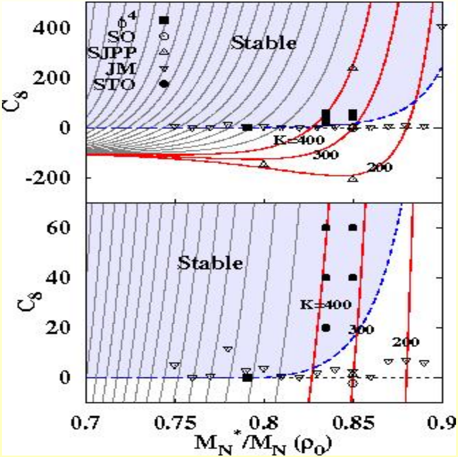
In the present treatment, we have five parameters, $g_\omega, g_\rho, C_4, C_6$ and C_8. Here we determine three parameters, g_ω, C_4 and C_6, in symmetric nuclear matter as functions of C_8 and the nucleon effective (Landau) mass $M_N^\star(\rho_0)$, by fitting the empirical saturation point, $(\rho_0, E_0/A)$, where E_0/A is the binding energy per nucleon at saturation density, $\rho_B = \rho_0$.

Parameters



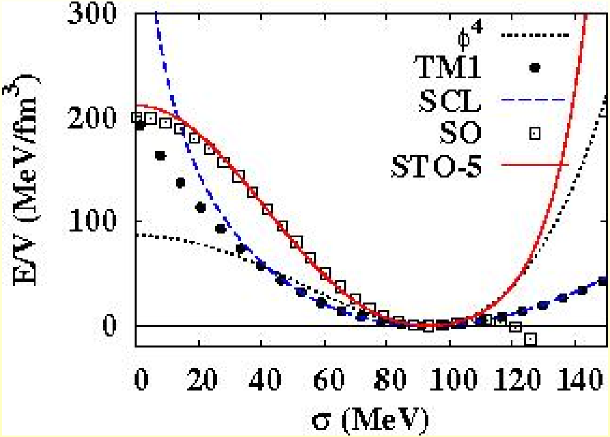
☞ C_8 values on the vacuum stability boundary

Stability



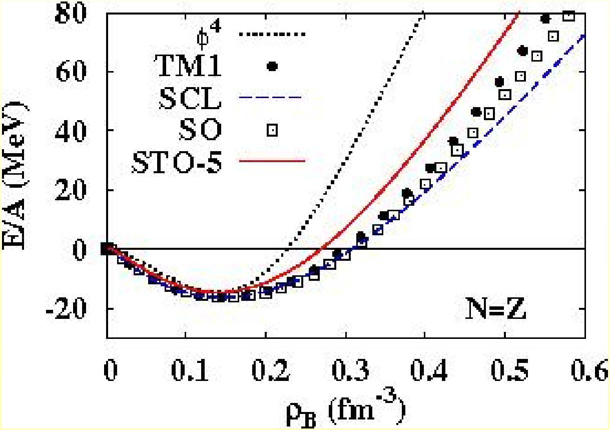
- ☞ Vacuum stability shaded area
- ☞ Incompressibility region

Chiral potential



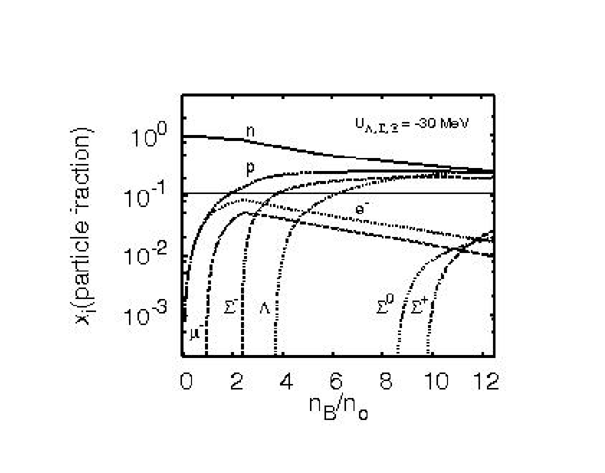
☞ Chiral potential as a function of σ; See Sahu et al., Physical Review C81 (2010) 014002

Equation of state



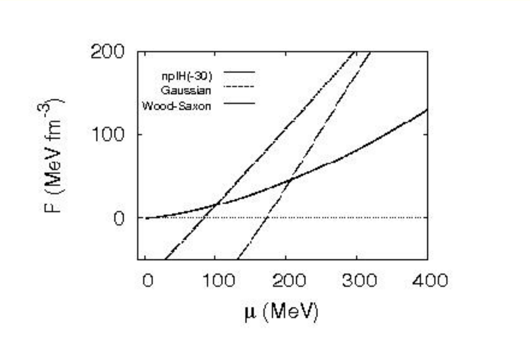
☞ See Sahu et al., Physical Review C81 (2010) 014002

Particle ratio in neutron star matter



- ☞ See Sahu et al.,Nuclear Physics A691 (2001) 439
- ☞ For one set type of model STO-V

Phase transition to quark matter



☞ See Sahu et al., Physics Letters B526 (2002) 19; Physical Review C66 (2002) 025802

☞ Assuming the first order phase transition near the high density and T$\rightarrow 0$ region

Naive dimensional analysis

The present STO model is a kind of effective field theory, contains higher order terms and is non-renormalizable. Then it would be valuable to examine the naturalness in the naive dimensional analysis (NDA). It is found that the loop contributions with the momentum cutoff $\Lambda \sim 1\ \mathrm{GeV}$ generate the following terms with dimensionless coefficients C_{lmnp} of order unity,

$$\mathcal{L}_{\rm int} \sim \sum_{l,m,n,p} \frac{C_{lmnp}}{m!n!p!} \left(\frac{\bar{\psi}\Gamma\psi}{f_\pi^2\Lambda}\right)^l \times \left(\frac{\varphi}{f_\pi}\right)^m \left(\frac{\omega}{f_\pi}\right)^n \left(\frac{\rho}{f_\pi}\right)^p (f_\pi\Lambda)^2 \ , \qquad (11)$$

where Γ denotes the γ and $\tau/2$ when necessary.

An effective theory having terms in Eq. (11) is considered to hold naturalness, when all the dimensionless coefficients C_{lmnp} are of order unity.

In the present effective Lagrangian, we obtain the following dimensionless coefficients,

$$
\begin{aligned}
C_{1100} &= \frac{f_\pi g_\sigma}{\Lambda} = \frac{M_N}{\Lambda} \sim 0.94 \ , \\
C_{1010} &= \frac{f_\pi g_\omega}{\Lambda} \sim 0.56 \ , \\
C_{1001} &= \frac{2 f_\pi g_\rho}{\Lambda} \sim 0.64 \ , \\
C_{0120} &= -\frac{2 g_{\sigma\omega}^2 f_\pi^2}{\Lambda^2} = -\frac{2 m_\omega^2}{\Lambda^2} \sim 1.2 \ , \\
C_{0220} &= \frac{2 g_{\sigma\omega}^2 f_\pi^2}{\Lambda^2} = \frac{2 m_\omega^2}{\Lambda^2} \sim 1.2 \ ,
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
C_{0300} &= \frac{f_\pi^2}{\Lambda^2}\, 3!\, \left(\frac{4}{3}C_6 - C_4\right) \sim -4\ , \\
C_{0400} &= \frac{f_\pi^2}{\Lambda^2}\, 4!\, \left(2C_8 - 2C_6 + \frac{1}{4}C_4\right) \sim 40\ , \\
C_{0500} &= \frac{f_\pi^2}{\Lambda^2}\, 5!\, \left(-4C_8 + C_6\right) \sim -280\ , \\
C_{0600} &= \frac{f_\pi^2}{\Lambda^2}\, 6!\, \left(3C_8 - \frac{1}{6}C_6\right) \sim 1200\ , \\
C_{0700} &= -\frac{f_\pi^2}{\Lambda^2}\, 7! C_8 \sim -2600\ , \\
C_{0800} &= \frac{f_\pi^2}{\Lambda^2}\, \frac{8!}{8} C_8 \sim 2600\ .
\end{aligned}
\tag{13}
$$

We find the results in STO-5, and adopt $\Lambda = 1~\mathrm{GeV}$. We find that the meson-nucleon and $\sigma\omega$ couplings are natural, but the self-interaction coefficients in σ are not natural.

Summary and Conclusion

- In this presentation, we have investigated the properties of nuclear matter and neutron star matter with quark gluon phase transition in the effective chiral model with σ^6 and σ^8 terms.
- The nucleon-vector meson coupling is found to be uniquely determined as a function of the effective mass at normal nuclear matter density, $Y(\rho_0) \equiv M_N^\star(\rho_0)/M_N$, and we have specified the region of stability in the $(Y(\rho_0), C_8)$ plane, where C_8 is the coefficient of the σ^8 term.
- We can find the parameter sets which satisfies the vacuum stability condition and moderate $K = (200 - 400)\ \mathrm{MeV}$ which is dominated by the nucleon effective mass.

- The obtained effective chiral model with higher order terms in σ is applied to neutron star matter, quark matter phase transitions and finite nuclei for the first time.
- We have also performed the naïve dimensional analysis (NDA) of the present model. Moderate K value of around 300 MeV requires the σ^8 coefficient $C_8 \gtrsim 20$ and this value corresponds to $C_{0800} \gtrsim 870$, and the model cannot hold naturalness.
- However, non-linear terms can give rise to three body forces, because three body forces is important at high density.
- This model can be used for the forthcoming experiment at FAIR(CBM).