# Universal Properties of the QCD Chiral Transition

## Results from Lattice QCD

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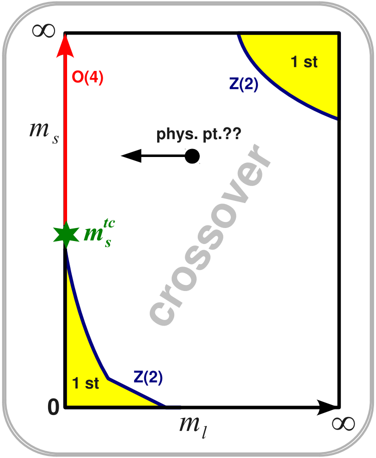
# A brief introduction



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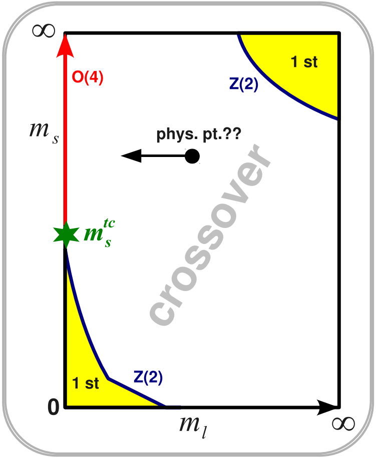


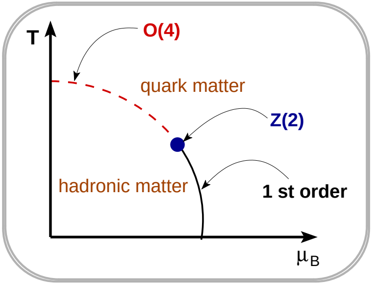

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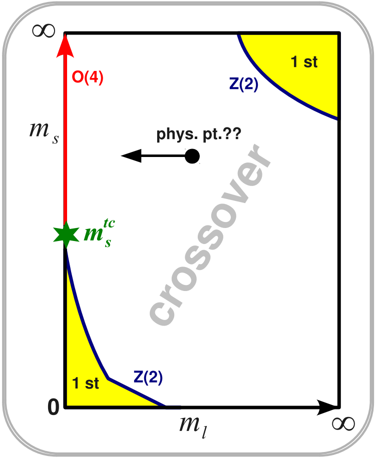


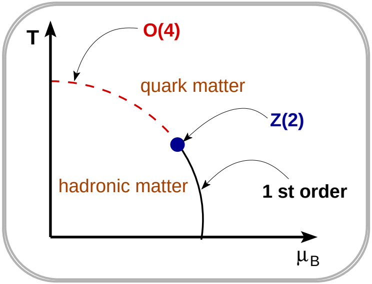


- ★ $N_f = 2$ QCD: 3 mass-less pions, $O(4)$
- ★ $a \neq 0$ staggered QCD: 1 mass-less pion, $O(2)$

$O(N)$

# Universal $O(N)$ scaling

- $$\frac{T}{V} \ln \mathcal{Z} = f_s(t,h) + f_{reg}(T,H,m_s)$$
- $$f_s(t,h) = h^{1+1/\delta} f_M(z)$$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c}$$

$$h = \frac{H}{h_0} = \frac{1}{h_0} \frac{m_l}{m_s}$$

★ $$\boldsymbol{z = t\, h^{-1/\beta\delta}}$$

★ $$\boldsymbol{z_0 = t_0^{-1} h_0^{1/\beta\delta}}$$

- **Chiral condensate**

$$M = h^{1/\delta} f_G(z)$$

- **Chiral susceptibility**

$$\chi_h = h^{1/\delta - 1} f_\chi(z)$$

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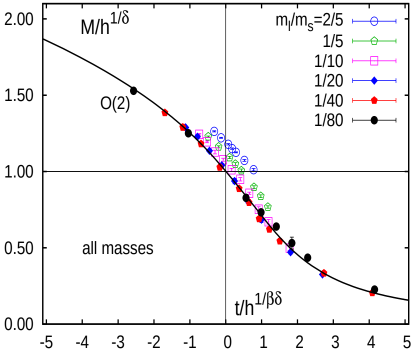
- $M(t=0, h=0) = 0$
- $M(t, h=0) = |t|^\beta$
- $M(t=0, h) = h^{1/\delta}$

# Scaling of the order parameter

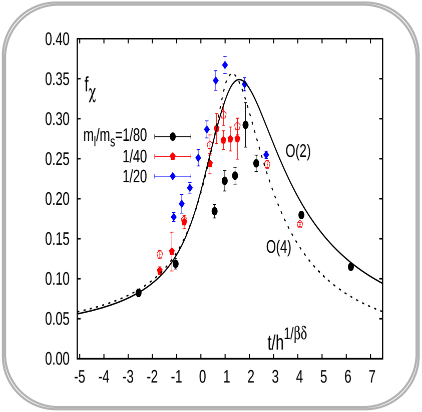



- p4fat3
- $N_\tau = 4 \times 32^3, 16^3, 8^3$
- physical $m_K$
- $m_\pi = 75 - 440$ MeV
- $M = m_s \left\langle \bar{\psi}\psi \right\rangle / T^4$
- fitted: $T_c$, $t_0$, $h_0$

★ scaling for: **$m_\pi \leq 150$ MeV**

Ejiri *et. al.*, PRD 80, 094505 (2009)

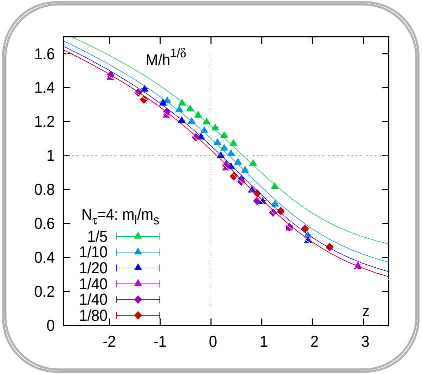
# Scaling of the chiral susceptibility




★ **no fitting**

Ejiri *et. al.*, PRD 80, 094505 (2009)

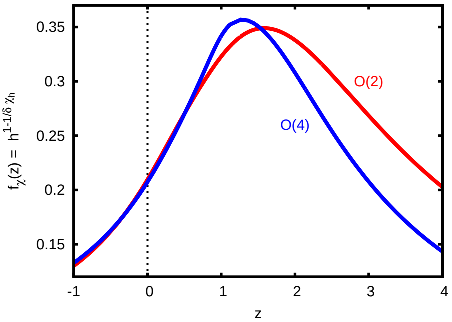
# Scaling violation




★ $m_\pi \gtrsim$ 150 Mev, scaling violation: $M = h^{1/\delta} f_G(z) + f_{reg}(T, H)$

$$f_{reg}(T, H) = a_t H \left(\frac{T - T_c}{T_c}\right) + b_1 H$$

# $T_{pc}$ and crossover

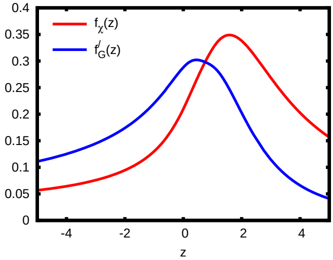



- $\chi_h = h^{1/\delta - 1} f_\chi(z)$
- $z = t\, h^{-1/\beta\delta}$

$$T_{pc}(H) = T_c \left( 1 + \frac{z_p}{z_0} H^{1/\beta\delta} \right)$$

# $T_{pc}$ and crossover

- only **2** relevant parameters (***h***, ***t***)
- only **3** susceptibilities




★ $\chi_h = h^{\frac{1-\delta}{\delta}} f_\chi(z)$

- chiral susceptibility

★ $\chi_{h,t} = h^{\frac{\beta-1}{\beta\delta}} f'_G(z)$

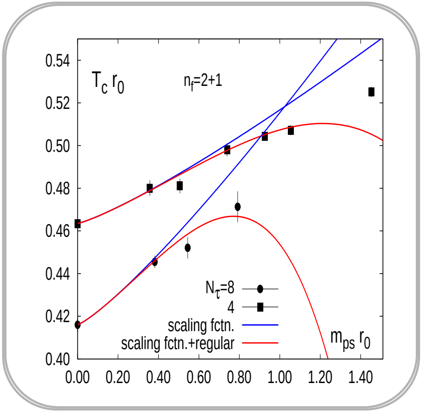
- mixed susceptibility ($\equiv$ infection pt. of M)

★ $\chi_t = h^{-\frac{\alpha}{\beta\delta}} f_t(z) + f_{reg}(T, H, m_s)$

- $c_v$, $\chi_4^B$, $\chi_L$ ($\equiv$ inflection pt. of $\epsilon$, $\chi_2^B$ $L$)

At most **3** $T_{pc}$

# $T_{pc}$ and crossover



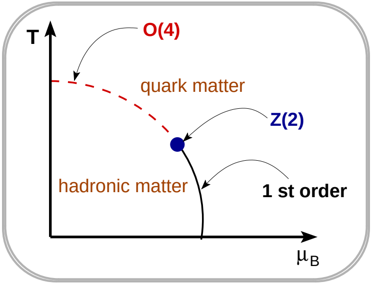

★ **no fitting**

# Universality for small $\mu_q$

★ **small $\mu_q$**

- no explicit $\chi$SM breaking by $\mu_q$

$$t = \frac{1}{t_0}\left[\frac{T - T_c}{T_c} + \kappa_q \left(\frac{\mu_q}{T}\right)^2\right]$$




$$\mu_q = (\mu_u + \mu_d)/2 = \mu_B/3$$

★ $t = 0$ critical line in the $T - \mu_q$ plane

**curvature**: $$\kappa_q \times \left(\frac{\mu_q}{T}\right)^2 = -\frac{T - T_c}{T_c}$$

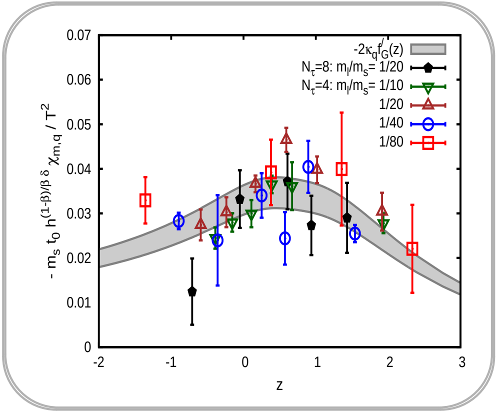
# Scaling and curvature of the chiral critical line

★ **mixed** susceptibility of the order parameter

$$\chi_{h,t} = \frac{\partial^2 f(h,t)}{\partial t \partial h}$$

$$\chi_{m,q} = \frac{2\kappa_q}{t_0} h^{\frac{\beta-1}{\beta\delta}} f'_G(z)$$

$$\chi_{m,q} = \frac{\partial^2 M}{\partial^2 \mu_q}$$

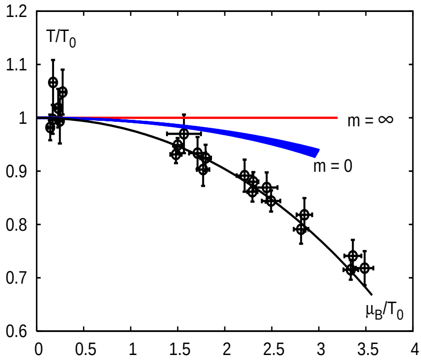



$$\kappa_q = \mathbf{0.059(5)}$$

★ $\kappa_B = 0.0066(6)$

Kaczmarek *et. al.*, arXiv:1011.3130

# Comparison with the freeze-out curve




$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \mathbf{0.0066}\left(\frac{\mu_B}{T}\right)^2 + \cdots$$

★ Freeze-out curve: Cleymans *et. al.*, PRC 73, 034905 (2006)

$$\frac{T_{fo}(\mu_B)}{T_c(0)} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

# Summary

- ★ QCD chiral transition belongs to $O(N)$ universality class: evidence from first principal Lattice QCD studies

- ★ Physical QCD may lie within the scaling region

- ★ Quantifying the width of the crossover region from universal scaling properties

- ★ Curvature of the chiral transition line in $T - \mu_B$ plane
  - small curvature, differs from the freeze-out curve