CNT QGP MEET 2015

HYDRODYNAMIC EVOLUTION OF QGP WITH SHADOWED GLAUBER MODEL

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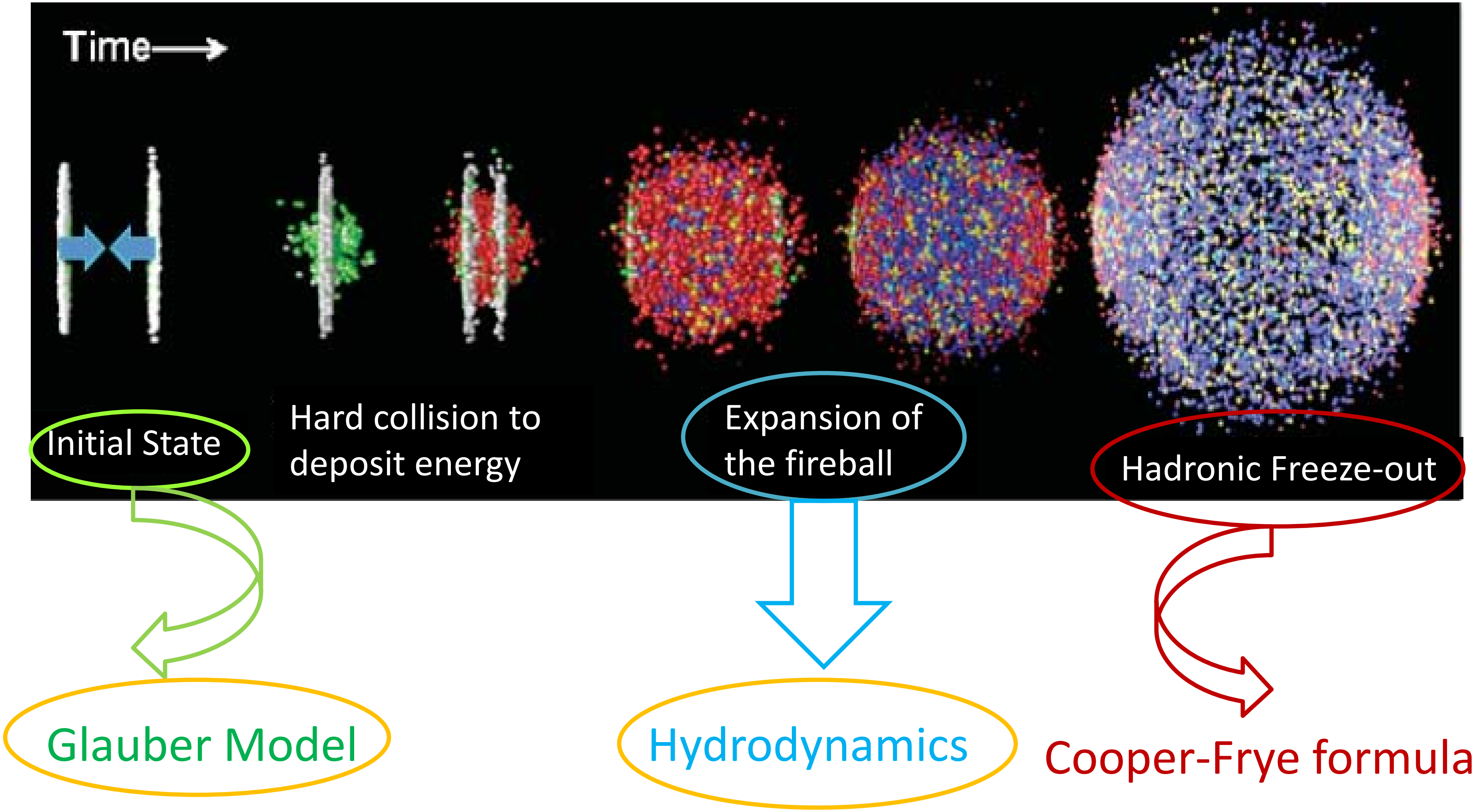
17th November, 2015

Outline

- ❑ Introduction
- ❑ Can we apply hydrodynamics in HICs?
- ❑ Code for solving hydrodynamic equations.
- ❑ Effects of shadowing of Glauber Model on hydrodynamic evolution.
- ❑ Results
- ❑ Summary

Heavy Ion Collision



- ❑ Our intension is to understand the initial state as well as the hydrodynamic evolution of the system.

Hydrodynamics

- “Hydrodynamics” is the theoretical framework for describing the motion of an expanding system.

- With specified initial conditions and the equation of state, the space-time evolution of the fluid can be directly derived from the dynamical equations.

Hydrodynamics in HICs?

- ❑ Mean free path << system size

- ❑ Collision rate between particles should be large compare to expansion rate to make the system thermalized(locally).

- ❑ For a typical heavy ion collision, system size ~ 10-15 fm and the mean free path is of the order of ~ 0.2-0.3 fm.

- ❑ Thus we can apply hydrodynamics for QGP in heavy ion collisions.

Assumptions

- For ultra-relativistic heavy ion collisions ($\sqrt{S_{NN}} = 2.76TeV$ for LHC), the system is dominated by gluons (in mid-rapidity region).
- QGP can be considered as net Baryonless fluid.
- Baryonic chemical potential is zero.
- Temperature is the only thermodynamic variable to quantify the system
- Viscosities of the system is assumed to be zero.

Ideal Hydrodynamics: Equations

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

$$\boxed{T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}}$$

$$\boxed{g^{\mu\nu} = (1, -1, -1, -1)} \quad \boxed{u^\mu = \gamma(1, \vec{v})}$$

$$\boxed{u^\mu u_\mu = \gamma^2(1 - \vec{v}^2) = 1}$$

ϵ and P are energy density and pressure of the system.

- For the consideration of baryonless fluid we neglect the equation

$$\boxed{\partial_\mu N^\mu = 0}$$

Ideal Hydrodynamics: Equations

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

$$\Downarrow$$

$$\begin{aligned}
\frac{\partial(\epsilon+P)\gamma^2 - P}{\partial t} + \frac{\partial(\epsilon+P)\gamma^2 v_x}{\partial x} + \frac{\partial(\epsilon+P)\gamma^2 v_y}{\partial y} + \frac{\partial(\epsilon+P)\gamma^2 v_z}{\partial z} &= 0 \\
\frac{\partial(\epsilon+P)\gamma^2 v_x}{\partial t} + \frac{\partial(\epsilon+P)\gamma^2 v_x^2 + p}{\partial x} + \frac{\partial(\epsilon+P)\gamma^2 v_x v_y}{\partial y} + \frac{\partial(\epsilon+P)\gamma^2 v_x v_z}{\partial z} &= 0 \\
\frac{\partial(\epsilon+P)\gamma^2 v_y}{\partial t} + \frac{\partial(\epsilon+P)\gamma^2 v_x v_y}{\partial x} + \frac{\partial(\epsilon+P)\gamma^2 v_y^2 + p}{\partial y} + \frac{\partial(\epsilon+P)\gamma^2 v_y v_z}{\partial z} &= 0 \\
\frac{\partial(\epsilon+P)\gamma^2 v_z}{\partial t} + \frac{\partial(\epsilon+P)\gamma^2 v_x v_z}{\partial x} + \frac{\partial(\epsilon+P)\gamma^2 v_y v_z}{\partial y} + \frac{\partial(\epsilon+P)\gamma^2 v_z^2 + P}{\partial z} &= 0
\end{aligned}$$

✓ These are coupled partial differential equation.

Bjorken hydrodynamics

- High-energy collisions, a flat dNch/dy , expected over a wide rapidity range.

- Bjorken hypothesized that thermodynamic variables are rapidity independent(mid rapidity range).

- Cartesian coordinate (t, x, y, z) **Transformation into** Milne coordinate (τ, x, y, η)

Substitute $\frac{d}{d\eta}(..) = 0$ Then Go back to cartesian coordinate

$$\frac{\partial}{\partial t}[(\epsilon+p)\gamma^2 - p] + \frac{\partial}{\partial x}[(\epsilon+p)\gamma^2 v_x] + + \frac{\partial}{\partial y}[(\epsilon+p)\gamma^2 v_y] + \frac{(\epsilon+p)\gamma^2}{t} = 0$$

$$\frac{\partial}{\partial t}[(\epsilon+p)\gamma^2 v_x] + \frac{\partial}{\partial x}[(\epsilon+p)\gamma^2 v_x^2 + p] + + \frac{\partial}{\partial y}[(\epsilon+p)\gamma^2 v_x v_y] + \frac{(\epsilon+p)\gamma^2 v_x}{t} = 0$$

$$\frac{\partial}{\partial t}[(\epsilon+p)\gamma^2 v_y] + \frac{\partial}{\partial x}[(\epsilon+p)\gamma^2 v_x v_y] + + \frac{\partial}{\partial y}[(\epsilon+p)\gamma^2 v_y^2 + p] + \frac{(\epsilon+p)\gamma^2 v_y}{t} = 0$$

These are (2+1)D ideal hydrodynamic equations

Solution

General form of the equations are

$$\boxed{\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + S = 0}$$

- These are called initial value flux conservative equations.
- Initial values of $\boxed{u, f, g, S}$ are to be supplied as inputs.
- Multi-dimensional Flux Corrected Transport (FCT) algorithm is used to solve iteratively.

Algorithm

- FCT algorithms can solve $\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = 0$, kind of equation

Ref: Numerical Recipes in FORTRAN

To solve we need to

- Discretize the space, time
- Define fluxes i.e f,g
- Define lower order fluxes (upwind / Lax-Friedrichs scheme) as well as higher order flux (Lax-Wendroff scheme)

- Upwind scheme:

$$u_{i,j}^{n+1} = u_{i,j}^{n} - \frac{\Delta t}{\Delta x}(f_{i,j}^{n} - f_{i-1,j}^{n}) - \frac{\Delta t}{\Delta y}(g_{i,j}^{n} - g_{i,j-1}^{n})$$

Where, $f_{i,j}^{n} = v_{i,j}^{n} u_{i,j}^{n}$

- Lower order schemes are good for shock-wave solution but have numerical dissipation $\Rightarrow$ conservation will be violated.

Algorithm

- Lax-Wendroff schemes:

$$u_{i,j}^{n+1} = u_{i,j}^{n} - \frac{\Delta t}{\Delta x}\left(f_{i+1/2,j}^{n+1/2} - f_{i-1/2,j}^{n+1/2}\right) - \frac{\Delta t}{\Delta y}\left(g_{i,j+1/2}^{n+1/2} - g_{i,j-1/2}^{n+1/2}\right)$$

- Higher order schemes do not have numerical dissipation but can not solve shock-wave problems.

- To make the solution stable we need to satisfy $\frac{v_{i,j}^{n}\Delta_t}{\Delta_x} \leq 1$

❑ This is known as Courant-Lewy-Friedrich (CFL) criteria or simply the Courant condition.

Algorithm

Ref : Steven T. Zalesak, J. of Computational Physics 31(1979)

- FCT do the job with both the schemes to get accurate solution.
- Steps are
  - Transported and diffusive solution is defined as

$$\boxed{u_{i,j}^{td} = u_{i,j}^{n} - \frac{\Delta t}{\Delta x}(f_{i+1/2,j}^{L} - f_{i-1/2,j}^{L}) - \frac{\Delta t}{\Delta y}(g_{i,j+1/2}^{L} - g_{i,j-1/2}^{L})}$$

  - Anti diffusive flux are defined as

$$\boxed{A_{i+1/2,j} = f_{i+1/2,j}^{H} - f_{i+1/2,j}^{L}} \qquad \boxed{B_{i,j+1/2} = g_{i,j+1/2}^{H} - g_{i,j+1/2}^{L}}$$

  - To limit the Anti-diffusion $\boxed{0 \leq C_{i+(1/2)} \leq 1}$ Process is too lengthy

$$\boxed{A_{i+1/2,j}^{C} = A_{i+1/2,j}C_{i+1/2,j}} \qquad \boxed{B_{i,j+1/2}^{C} = B_{i,j+1/2}D_{i,j+1/2}}$$

Algorithm

- Solution to be

$$u_{i,j}^{n+1} = u_{i,j}^{td} - \frac{\Delta t}{\Delta x}(A_{i+1/2,j}^{C} - A_{i-1/2,j}^{c}) - \frac{\Delta t}{\Delta y}(B_{i,j+1/2}^{C} - B_{i,j-1/2}^{C})$$

- But we have solve equations like

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + S = 0$$

- Solution:

$$u_{i,j}^{n+1} = u_{i,j}^{n+1} - dt * (S_{i,j}^{n})$$

❑ From the solution we can extract thermodynamic variables easily

Ref : Richke et al, Nuclear Physics A 595 (1995)

Testing the code

Gubser solution:

Ref : P. Houvinen et al, Computer Physics Communication 185(2014)

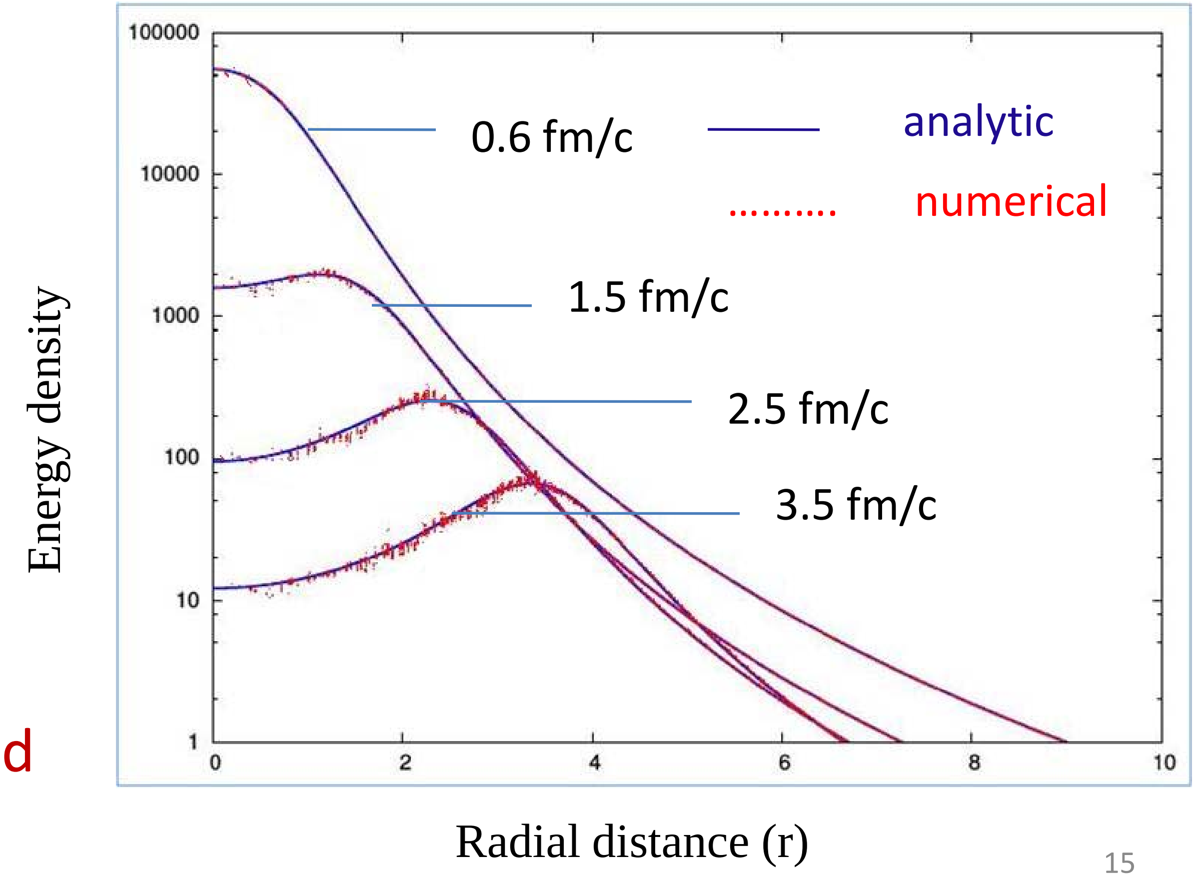
$$\epsilon = \frac{\epsilon_0 (2q)^{8/3}}{\tau^{4/3}} \left[1 + 2q^2(\tau^2 + r_T^2) + q^4(\tau^2 - r_T^2)^2\right]^{4/3}$$

$$v_x = \frac{2\tau x}{1+\tau^2+r^2}$$

$$v_y = \frac{2\tau y}{1+\tau^2+r^2}$$

$$P = \frac{\epsilon}{3}$$

- Gubser result is successfully reproduced by the code.



Input of the code for HICs

- ❑ Thermalization time ,here for RHIC energy , $\boxed{\tau_0 = 0.6 fm/c}$
- ❑ Initial energy density from Optical Glauber Model or Monte-Carlo Glauber Model (will be discussed later).
- ❑ Equation of state, $\boxed{P = \frac{\epsilon}{3}}$. Lattice QCD EoS will be used later.
- ❑ Transverse velocities of expansion were taken to be zero.
- ❑ Boundary Condition: Open boundary condition.

$$\mathcal{A}(x + \Delta x) = \mathcal{A}(x)$$

$$\mathcal{A}(x + 2\Delta x) = \mathcal{A}(x)$$

Initial conditions

- Important to study heavy ion physics.
- Two types of initial conditions are available to get fairly good results HICs.
  - Glauber Model (our main interest)
  - CGC based IP-Glasma Model.

▪Glauber Model :

Characterize by number of wounded nucleons and number of binary collisions

Optical Glauber Model :

a) Supplies smooth energy density profile.
b) Fails to study fluctuations where positions of individual nucleons are relevant.

Monte-Carlo Glauber Model :

a) Supplies fluctuating energy density profile.
b) Quantum fluctuations of positions of the individual nucleons are taken into account.

Optical Glauber model: coding

Input of the code

Nucleon density: Woods-Saxon type without any deformation

$$\rho_A(b,x,y,z) = \frac{\rho_0}{1+\exp(\frac{r_1'-R_A}{\delta})}$$

$$\rho_B(b,x,y,z) = \frac{\rho_0}{1+\exp(\frac{r_2'-R_B}{\delta})}$$

where, $r_1' = \sqrt{(x+b/2)^2+y^2+z^2}$ and $r_2' = \sqrt{(x-b/2)^2+y^2+z^2}$

Au+Au collision

ρ_A	rho_B	R_A	R_B	δ_A	δ_B
0.169	0.169	6.62	6.62	0.54	0.54

- b is impact parameter

σ_{NN}= 42 mb

Optical Glauber model: coding

Thickness function:

$$T_A(x,y) = \int \rho_A(x,y,z)dz$$

$$T_B(x,y) = \int \rho_B(x,y,z)dz$$

$$T_{AB}(x,y) = \int \rho_A(x,y,z)\rho_B(x,y,z)dz$$

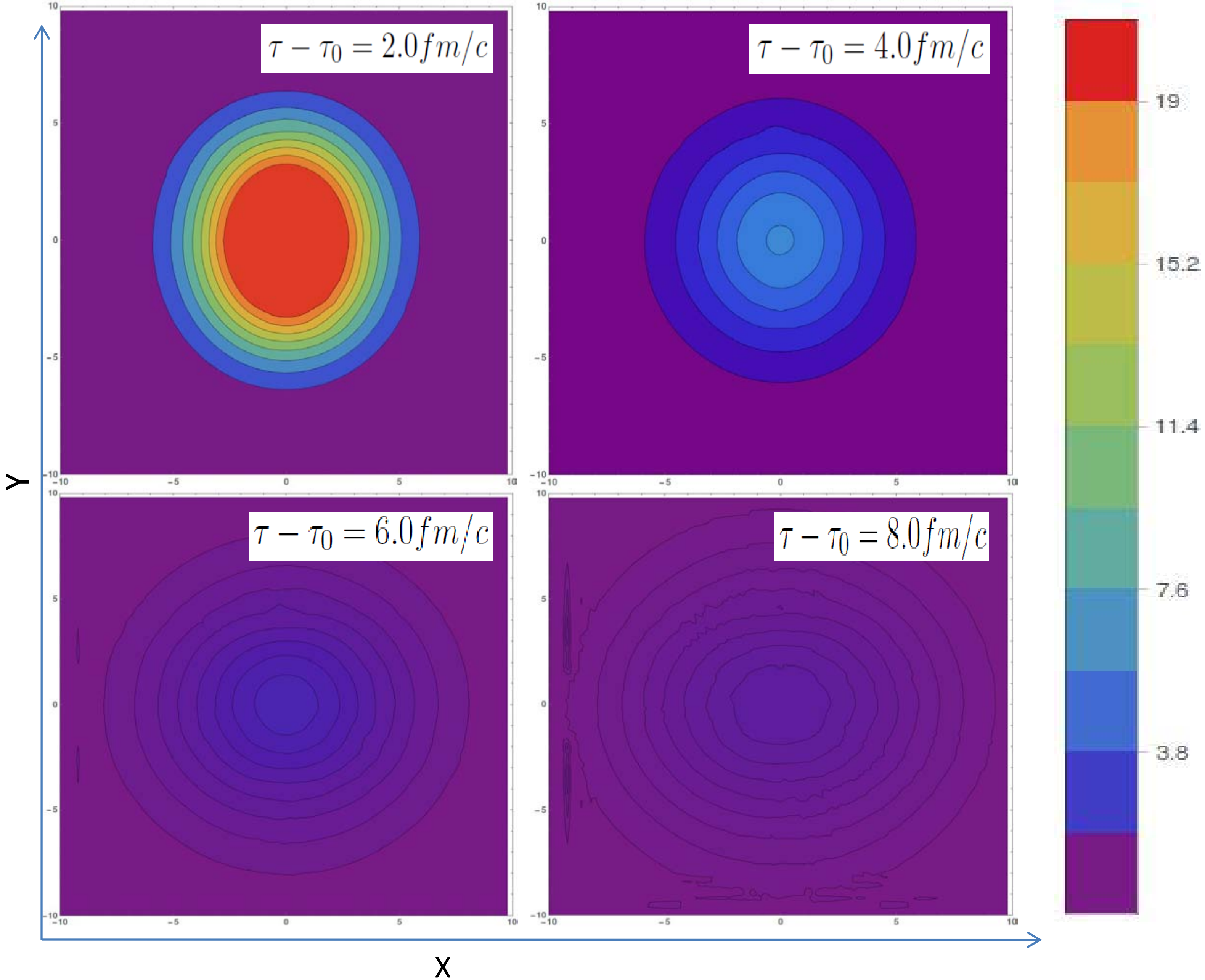
$$N_{part}(x,y) = T_A(x,y)(1-e^{-\sigma_{NN}T_B(x,y)}) + T_B(x,y)(1-e^{-\sigma_{NN}T_A(x,y)})$$

$$N_{coll}(x,y) = \sigma_{NN} TAB(x,y)$$

$$\epsilon(x,y) = \epsilon_0[(1-f)N_{part}(x,y) + fN_{coll}(x,y)]$$

- f = hardness parameter.

Optical Glauber : Results



Monte-Carlo Glauber Model

- Individual nucleon positions are considered.
- Nucleons (for nucleus A and B) are generated by Monte-Carlo random number generator.
- Sampled from Woods-Saxon type of distribution.
  - Distance between a nucleon of A to a nucleon of B

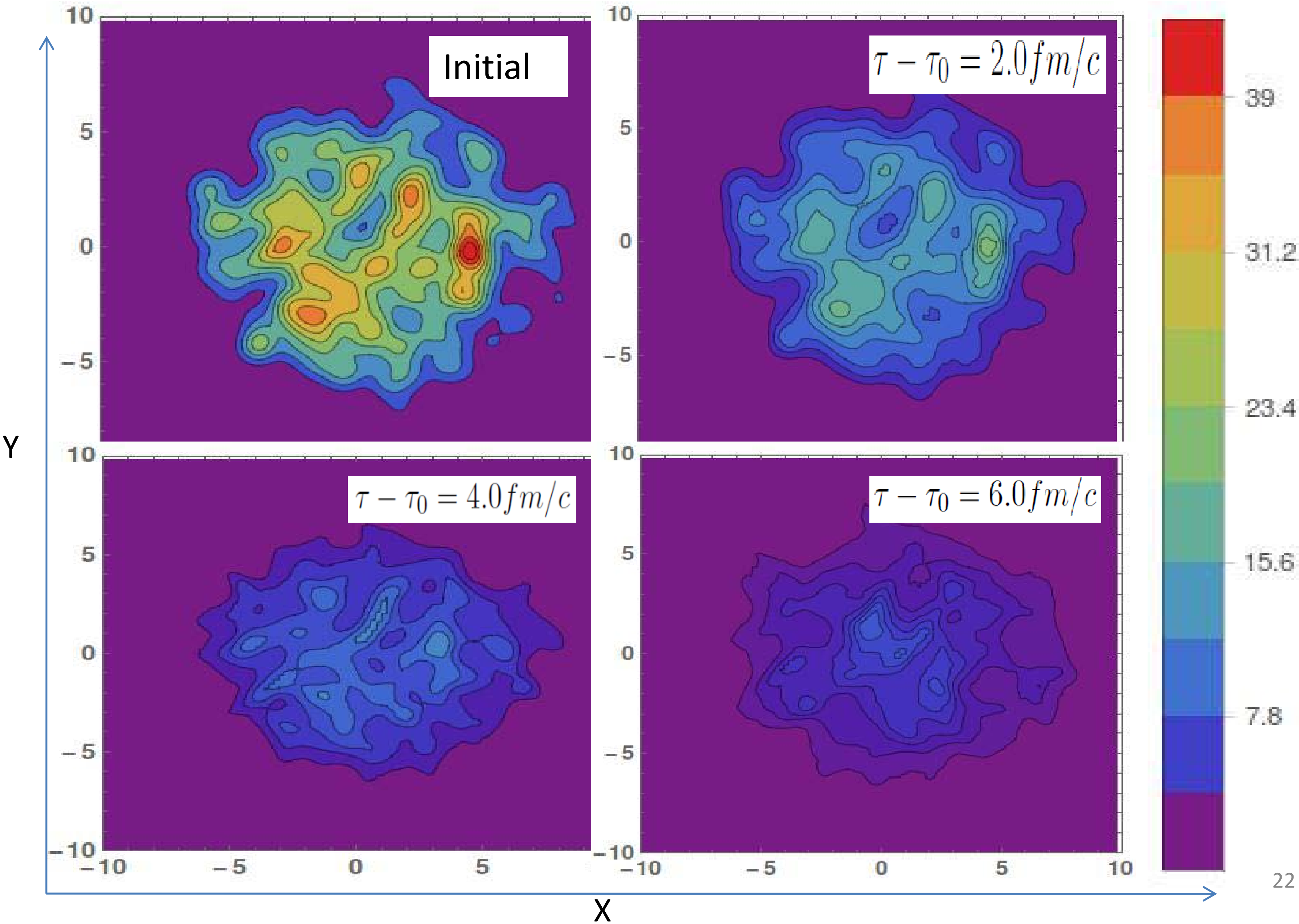
$$r_{ij}^{AB} = \sqrt{(x_i^A - x_j^B)^2 + (y_i^A - y_j^B)^2}$$

  - Criteria for collision $r_{ij}^{AB} \leq \sqrt{\frac{\sigma_{NN}}{\pi}}$
  - Energy deposition for ith source at position (x_i, y_i)

$$\epsilon_i(x, y) = \frac{\epsilon_0}{2\pi\sigma^2} e^{-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma^2}}$$

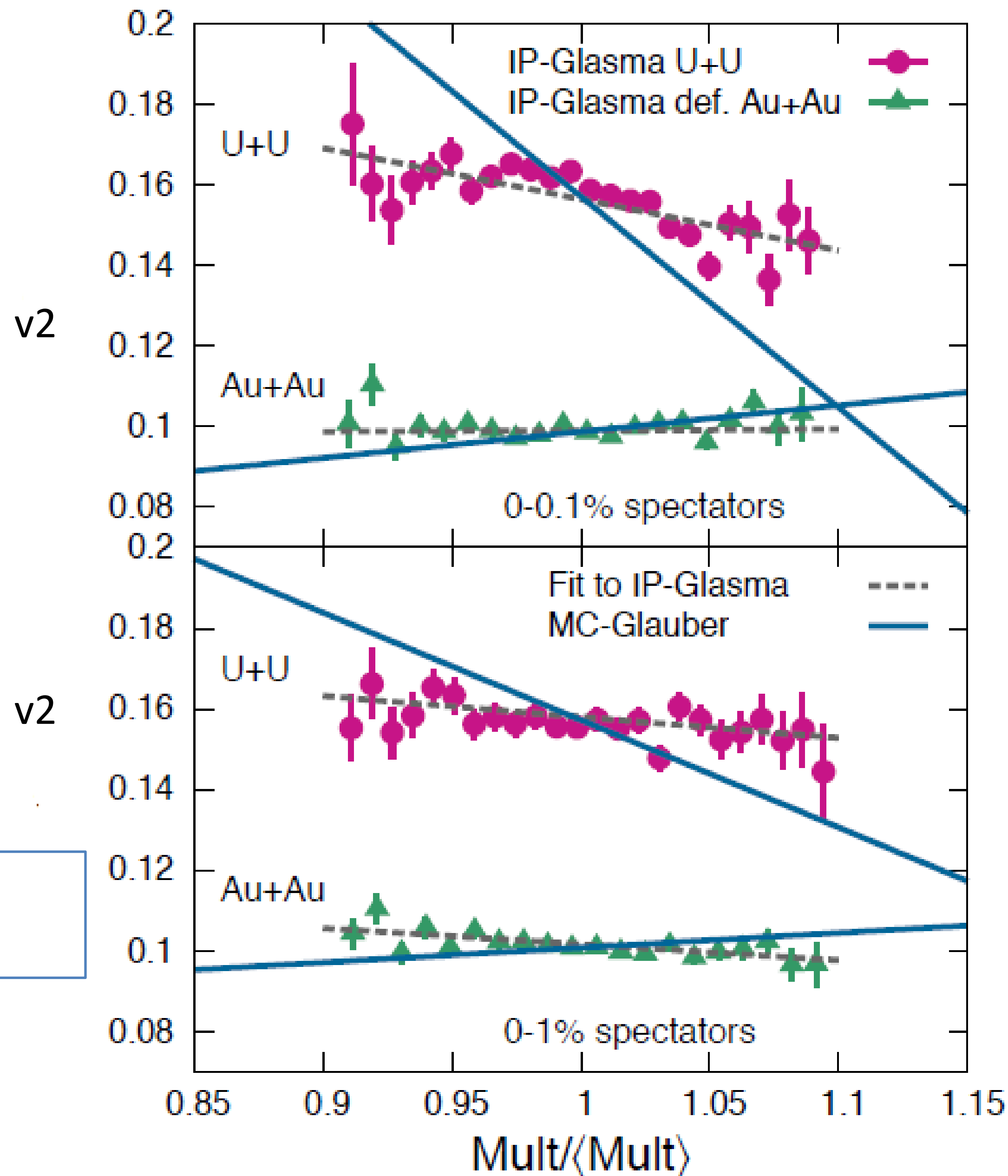
$$\sigma = 0.6 \text{ fm}$$

MCG: Results


Initial
$\tau - \tau_0 = 2.0 fm/c$
$\tau - \tau_0 = 4.0 fm/c$
$\tau - \tau_0 = 6.0 fm/c$
10
5
0
−5
−10
−10
−5
0
5
10
39
31.2
23.4
15.6
7.8
Y
X

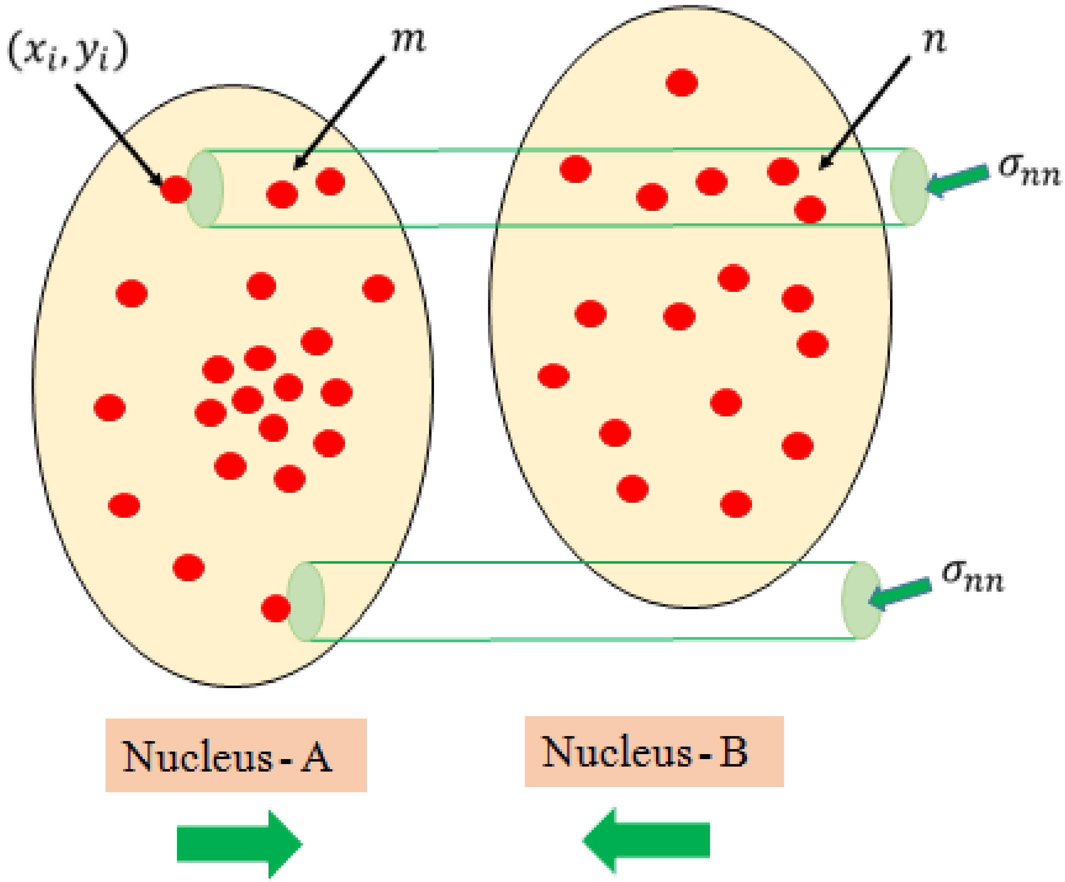
Glauber model : Drawback

❑ MCGM can not explain these plots for highly central events.



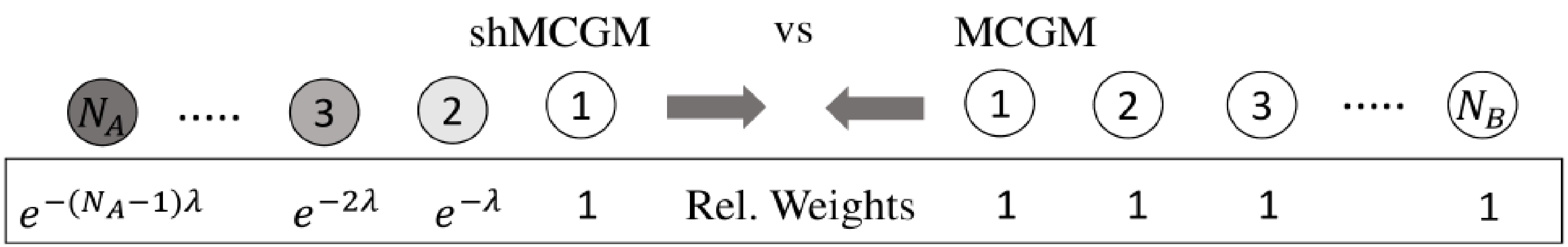
Ref : **PRC 89, 064908 (2014)**
Bjorn Schenke et al

Shadowing effects on Glauber Model



- ❑ All the collisions are not treated equally.
- ❑ A nucleon staying behind another nucleon will be shadowed(in cross section sense) by the front one.

Shadowing effects on Glauber Model



- ➢ N_{part} and N_{coll} gets modified.
- ➢ Energy density will also change

❑ We use simple suppression factor as $S(n, \lambda) = e^{-n\lambda}$

✓ n is the number of nucleons ahead of a nucleon

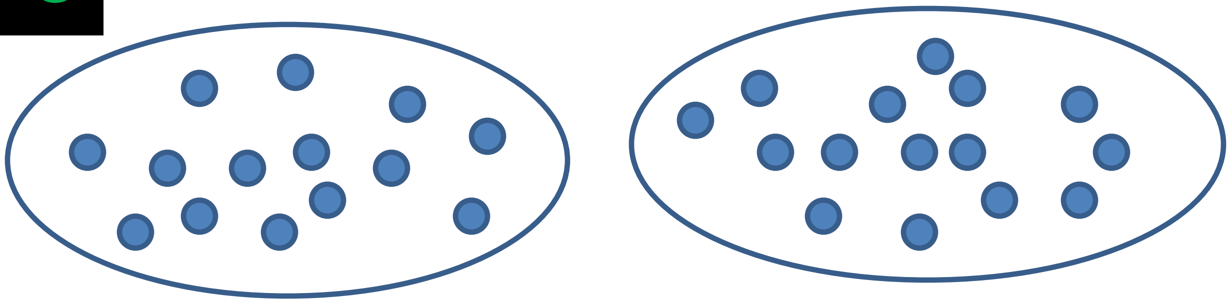
λ Is the shadow parameter fit to experimental observations.

❖ This effect can also be introduced on Optical Glauber Model.

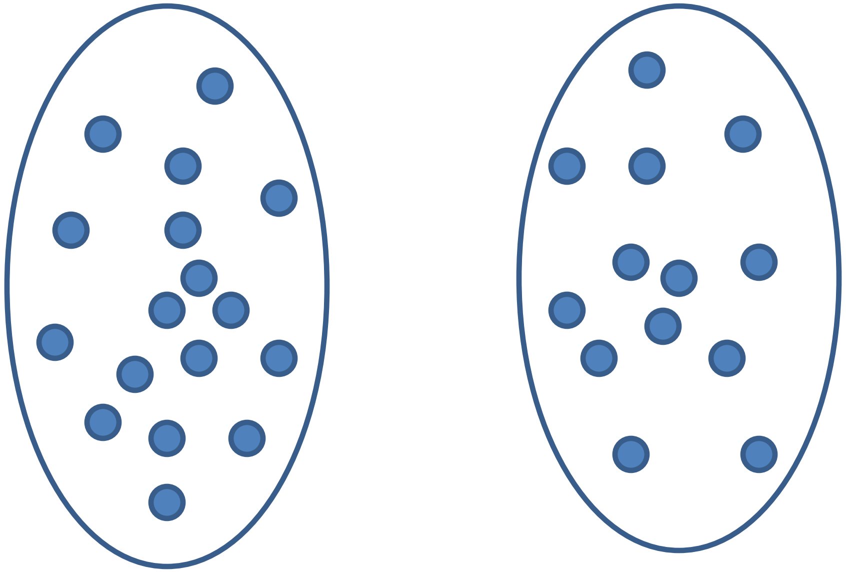
❖ Unlike MCGM, the nucleon density is continuous .

High multiplicity: Configurations

U+U



Tip-Tip configuration



Body-Body configuration

MCGM

$$N_{coll}^{TT} > N_{coll}^{BB}$$

$$N_{part}^{TT} = N_{part}^{BB}$$

$$\epsilon_2^{TT} < \epsilon_2^{BB}$$

shMCGM

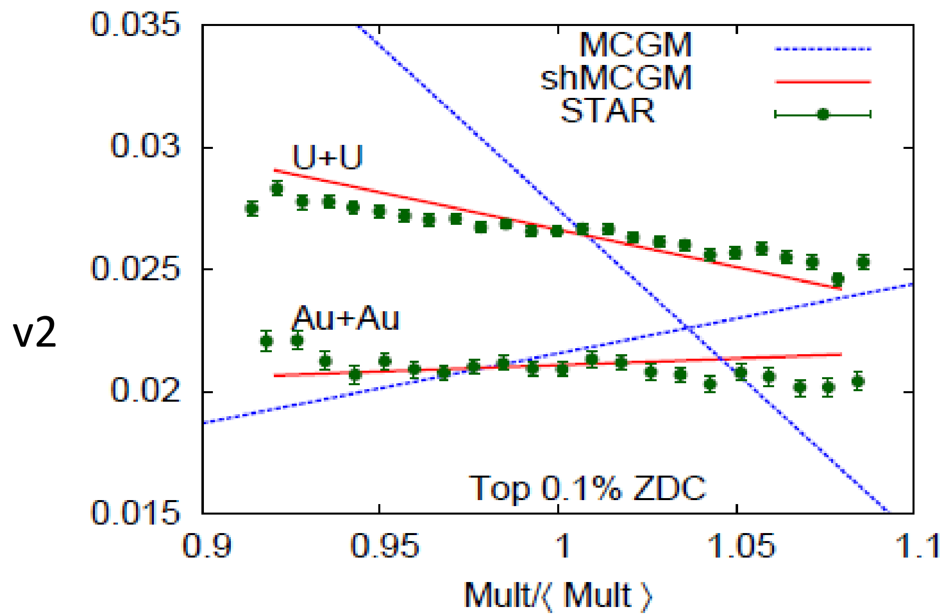
$$N_{coll}^{TT} \approx N_{coll}^{BB}$$

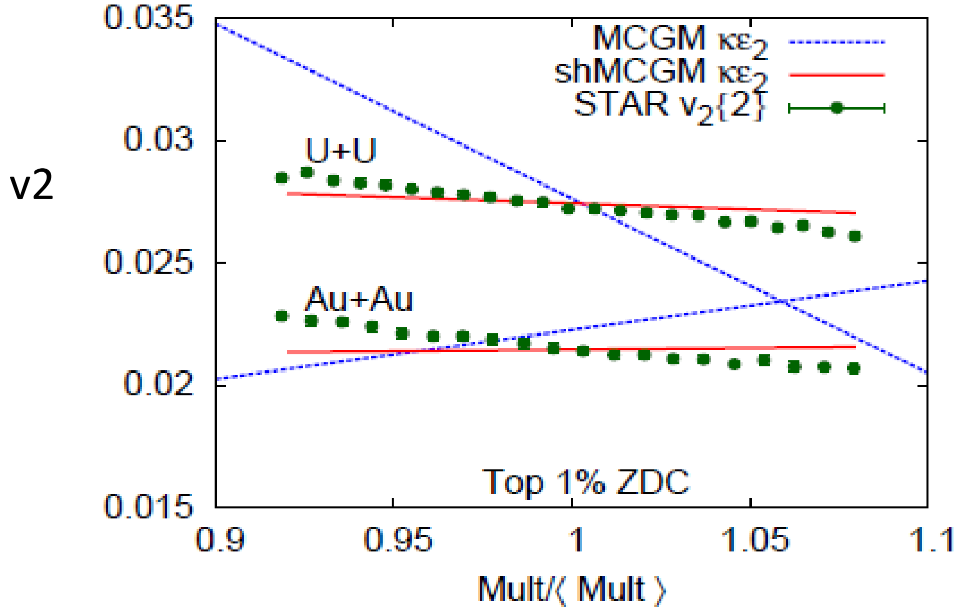
$$N_{part}^{TT} = N_{part}^{BB}$$

Results: shMCGM

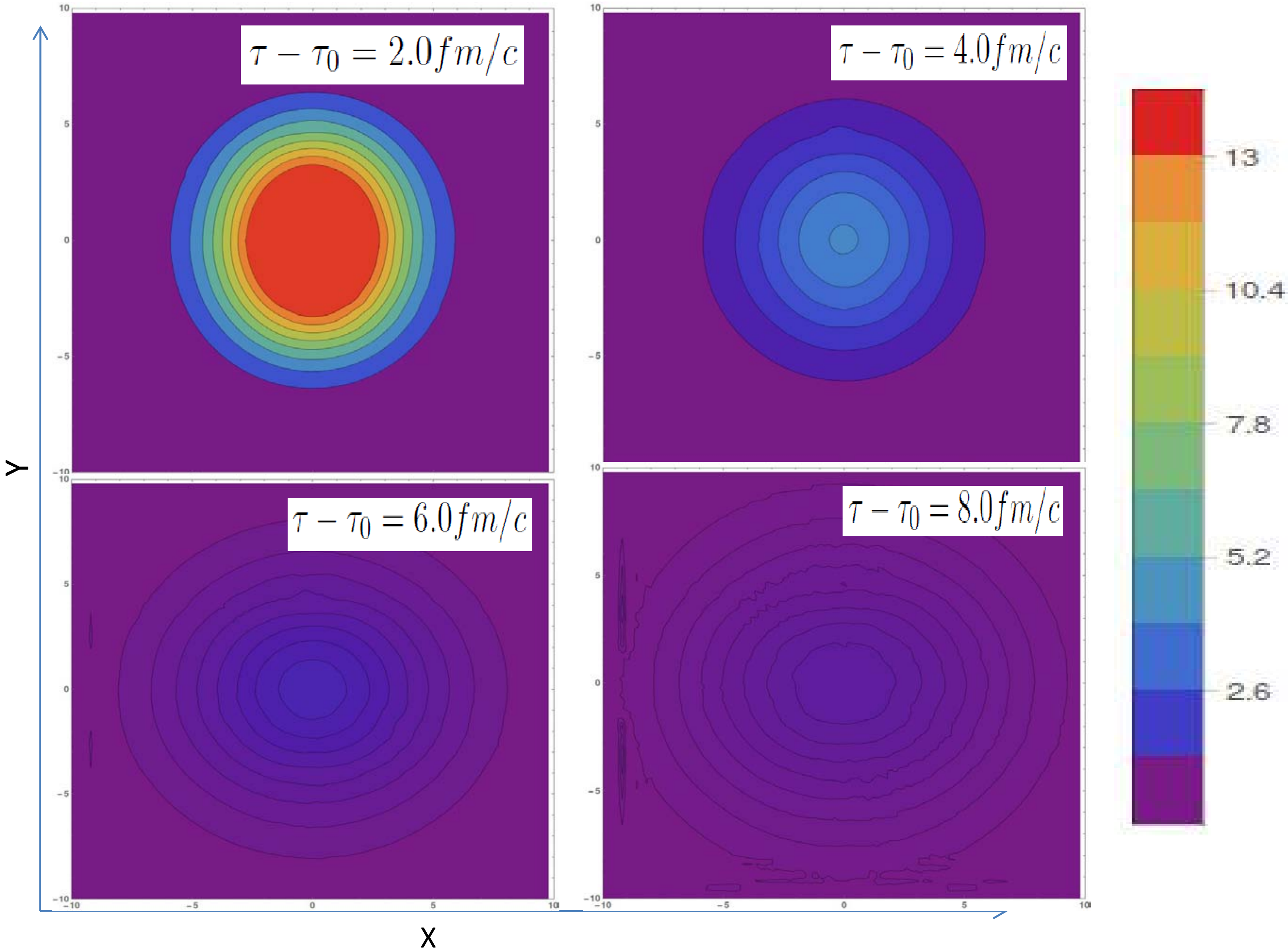
Ref : **[arXiv:1510.01311]**

shMCGM explains the data well





Shadowed Optical Glauber: Results



Results: Shadowed vs Unshadowed

- Impact parameter b = 7 fm.
- Shadow parameter λ = 0.1
- Energy density for central contour values are considered.

Glauber	$\tau - \tau_0 = 2.0 fm/c$	$\tau - \tau_0 = 4.0 fm/c$	$\tau - \tau_0 = 6.0 fm/c$	$\tau - \tau_0 = 8.0 fm/c$
Optical Glauber	20.12 GeV/fm^3	5.96 GeV/fm^3	1.47 GeV/fm^3	0.37 GeV/fm^3
Optical Glauber with shadowing	14.6 GeV/fm^3	4.47 GeV/fm^3	1.21 GeV/fm^3	0.24 GeV/fm^3

Summary

- ❑ Numerical code has been developed to solve hydrodynamic equations.
- ❑ Results of code have been contrasted with others.
- ❑ Effects of shadowing on initial conditions have been incorporated.
- ❑ Effects of shadowing are found to be very important.
- ❑ (3+1)D code with non-zero baryonic chemical potential is under development.

Collaborators

- Jan-e Alam
- Sandeep Chatterjee
- Snigdha Ghosh
- Sushant Kr. Singh
- Sourav Sarkar
- Golam Sarwar

Thank You