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Stochastic dynamical model of fission

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Stochastic dynamical model of fission

We discuss only stochastic model, not the "Dynamical cluster decay model" of Greiner,Gupta (Int.J.Mod.Phys.E3,supp01(1994)3350)

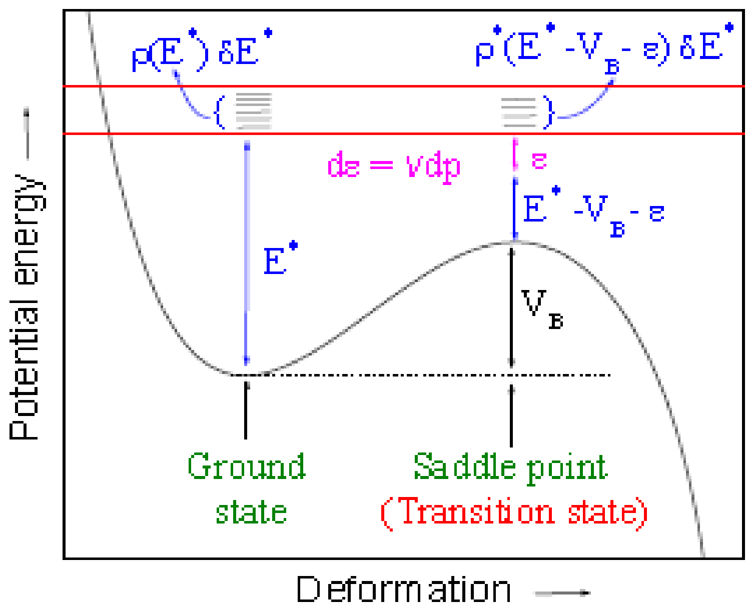
How does a nucleus undergo fission, EX> fission barrier

For EX< fission barrier, q.m. tunneling, not discussed here

Statistical model of Bohr and Wheeler
(N. Bohr and J.A. Wheeler, 1939) Phys.Rev.56(1939)426

Statistical → Full phase space equally accessible for an equilibrated system

Fission probability will depend upon the phase space available at the saddle point (the transition point)



$\rho(E^*)$ = density of states at ground state

No. of states between E^* and $E^*+\delta E^*$ = $\rho(E^*)\delta E^*$

Choose an ensemble such that there is only one nucleus in each state

No. of nucleus in ground state=$\rho(E^*)\delta E^*$

Collective kinetic energy=ε; collective speed=v ; $d\varepsilon$ =vdp

No. of collective states with momentum between p and p+dp and over one unit of length along fission direction
= $1\times dp/h = dp/h$

Excitation energy at saddle = $E^* - V_B - \varepsilon$

No. of nuclei at saddle over a distance v
= $v\times dp\times\rho^*(E^*-V_B-\varepsilon)\delta E^*/h$

Number of nuclei at saddle over a distance v
= v dp $\rho^*(E^*-V_B-\varepsilon)\delta E^*/h$

Total number crossing the saddle per unit time
= $\delta E^* \int v\,dp\,\rho^*(E^*-V_B-\varepsilon)/h$
= $\delta E^* \int d\varepsilon\, \rho^*(E^*-V_B-\varepsilon)\,/h$

Fission probability/time=r
=$[\delta E^* \int d\varepsilon\rho^*(E^*-V_B-\varepsilon)\,/h\,]/\,\rho(E^*)\delta E^*$
= $[\int d\varepsilon\rho^*(E^*-V_B-\varepsilon)]\,/h\,\rho(E^*)$

From radioactive decay law, fission life-time $\tau = 1/r$

Fission width $\Gamma_{BW}=\hbar/\tau=\hbar r$
= $(1/2\pi\,\rho(E^*))\int d\varepsilon\rho^*(E^*-V_B-\varepsilon)$

$$\Gamma_{BW} = \frac{1}{2\pi\rho(E^*)}\int_0^{E^*-V_B}\rho^*(E^*-V_B-\varepsilon)d\varepsilon$$

Approximate form: (Assume little "a" and "I" shape independent)

Level density (Fermi gas): combinatorial, Bohr-Mottelson, Nuclear Structure, Vol.I

$$\rho(E^*,l) = \frac{2l+1}{24}\left(\frac{\hbar^2}{2I}\right)^{\frac{3}{2}}\frac{\sqrt{a}}{E^{*2}}e^{2\sqrt{aE^*}} \approx \frac{e^{2\sqrt{aE^*}}}{E^{*2}}$$

$$\Gamma_{BW} = \frac{1}{2\pi}\int_0^{E^*-V_B}\frac{E^{*2}}{(E^*-V_B-\varepsilon)^2}e^{2\sqrt{a(E^*-V_B-\varepsilon)}-2\sqrt{aE^*}}d\varepsilon$$

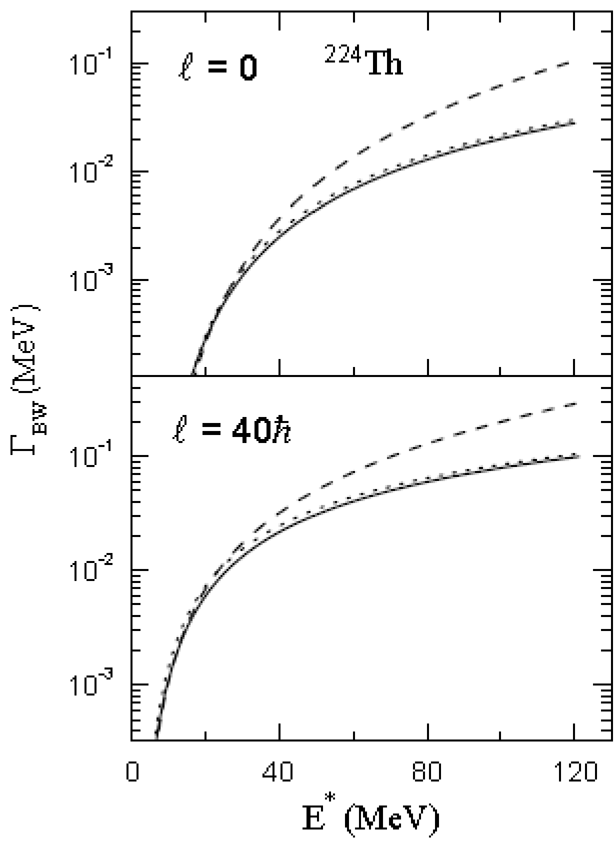
For E*>>V_B

$$\frac{E^*}{E^* - V_B - \varepsilon} \approx 1$$ Note: when ε=E*-V_B , integrand very small

$$\Gamma_{BW} \approx \frac{1}{2\pi} \int_0^{E^*-V_B} e^{2\sqrt{a(E^*-V_B-\varepsilon)}-2\sqrt{aE^*}} d\varepsilon$$

$$\sqrt{a(E^* - V_B - \varepsilon)} = \sqrt{aE^*} - \frac{V_B + \varepsilon}{2}\sqrt{\frac{a}{E^*}} = \sqrt{aE^*} - \frac{V_B + \varepsilon}{2T}$$

$$\Gamma_{BW} = \frac{T}{2\pi} e^{-\frac{V_B}{T}}$$



Sadhukhan,Pal:PRC79,064606(2009)

Strutinsky's correction (Phys.Lett.47B(1973)121)

We have missed out something

We have included collective phase space at saddle
But we have not accounted for the collective phase space at ground state

Assume an oscillator potential near ground state = $m\omega^2 x^2/2$
Distribution function (Boltzman)= $\exp[-(p^2/2m + m\omega^2 x^2/2)/T]$

$\Delta x = \sqrt{(2\pi T/m\omega^2)}$
$\Delta p = \sqrt{(2\pi m T)}$

Collective phase space factor at g.s. $=\Delta x \Delta p/h = T/\hbar\omega$ (to be multiplied with $\rho(E^*)$ in denominator)

Aproximate Gaussian by rectangle in obtaining $\Delta x, \Delta p$

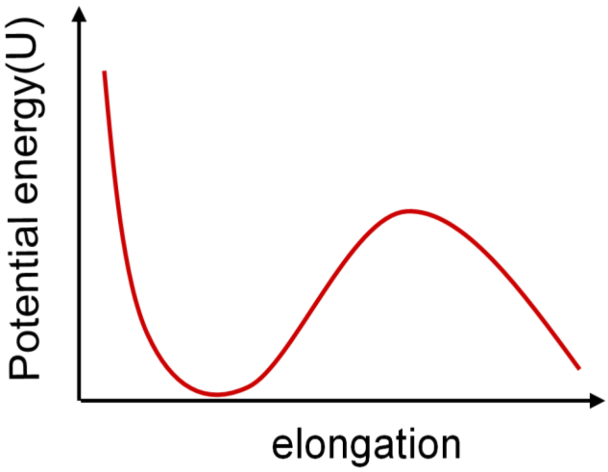
$$= \frac{\hbar\omega_g}{2\pi} e^{-V_B/T}$$

Strutinsky corrected

Crucial assumption in BW$\rightarrow$ equilibrium everywhere

True if flux across fission barrier is very small

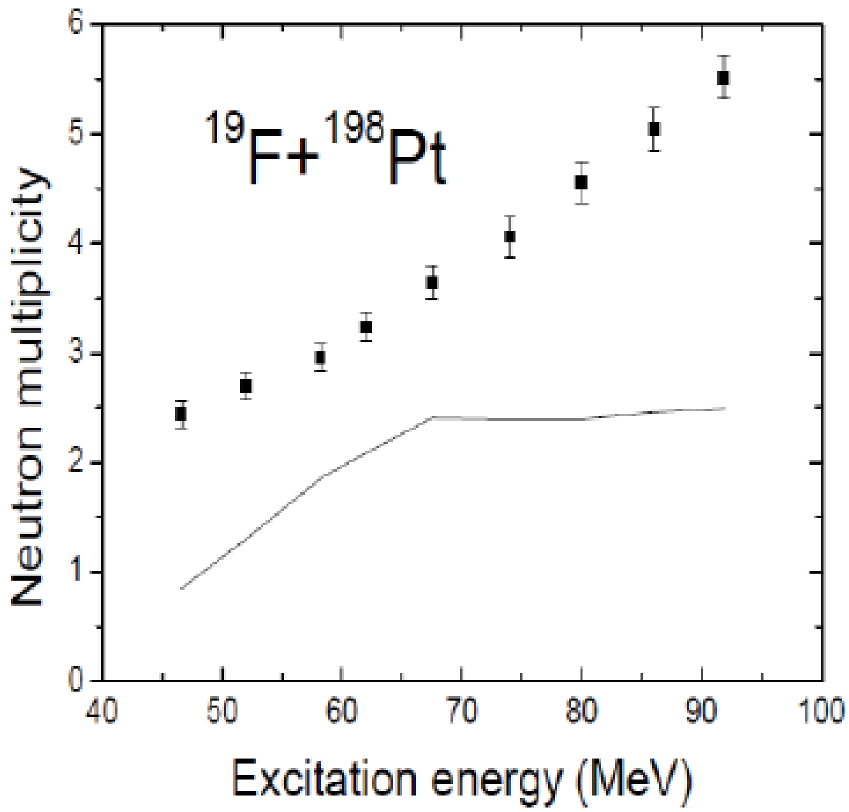
Fission width is very small $\rightarrow$ V_B>>T



If not (e.g. heavy ion reactions), not enough nuclei near barrier after initial crossing

To maintain steady flux at saddle, fission dynamics to be considered

Dynamical model



Expt. $\rightarrow$ V. Singh et al. Phys. Rev.C86(2012)014609

Line $\rightarrow$ Stat. model with Bohr-Wheeler width

Bohr-Wheeler over predicts fission rate

Fission is slower than predicted by Bohr-Wheeler theory

Collective dynamics of interacting particles-> dissipative

When fission is slower, more time for particle evaporation

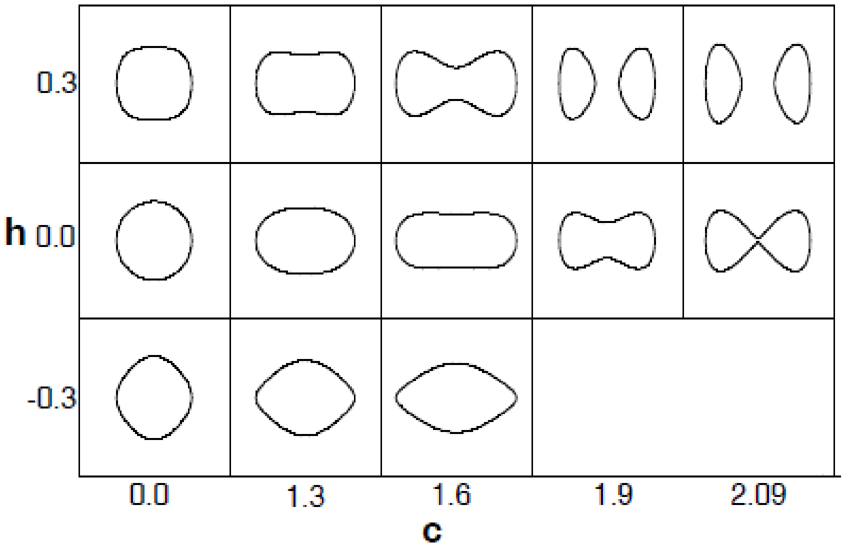
Fission $\Rightarrow$ shape evolution$\Rightarrow$ shape variables $\Rightarrow$ dynamical coordinates

Elongation (c)

Neck (h)

Asymmetry (α)

Brack (funny hill)



Nucleus$\Rightarrow$ 3A coordinates

->a few shape (collective) variables(x)

+ many intrinsic (almost 3A) coordinates (ξ)

Consider effect of V_{int} on H_{intr} as a first-order perturbation and take average over all intrinsic states (Linear Response Theory)

$d\langle H_{intr}\rangle/dt \propto (dX/dt)^2 \;\Rightarrow$ energy is pumped into intrinsic system **(heating)**

energy is lost from collective motion H_{coll} **(dissipative energy loss)**

<u>*Eqn. of motion of collective coordinates averaged over intrinsic states*</u>

$d\langle P\rangle/dt = -(dU/dX) - \eta\, (d\langle X\rangle/dt)$

Gives average trajectory in deformation (collective coordinate) space

- H.Hofmann & P.J.Siemens, Nucl. Phys. A 257 (1976)165
- S.E. Koonin & J.Randrup, Nucl. Phys. A 289 (1977) 475

  <u>*Stochastic dynamics*</u>*:*

- Consider an ensemble of fissioning compound nuclei.
- Shape evolution not same for all CN *(Had it been same, all CN would have reached the saddle point simultaneously and we would not have the law of radioactive decay, but same life-time for all CN).*
- Some reach saddle point earlier, some later.

- For a given time interval, average trajectory may or may not cross saddle. In reality, some trajectories cross saddle, some do not, the ratio gives the fission probability
- Average trajectory not of much use in fission dynamics. We need to trace individual trajectories.
- **We need observables averaged over many trajectories and not observables for the average trajectory**
- **We need eqn. of motion of individual trajectories**

  $H_{tot} = H_{coll}(x) + H_{intr}(\xi) + V_{int}(x,\xi)$

  The force on the collective dynamics due to $V_{int}(X,\xi)$ is random in nature essentially due to the large number of intrinsic degrees of freedom (ξ)

  Force= $\langle$Force$\rangle$ +fluctuation (R)

  $dP/dt= -(dU/dX)-\eta(dX/dt) +R(t)$

  Langevin equation of motion

  Fission->Brownian motion of a heavy particle in a viscous heat bath

Collective dynamics (large inertia) ->Brownian particle

$\langle R(t) \rangle = 0$

$\langle R(t)R(t') \rangle = 2D\delta(t-t')$

R(t) is assumed to follow a Gaussian distribution

Fluctuation-dissipation theorem: $D=\eta T$

Markovian Process (zero memory time) assumed

<u>How to solve a stochastic eqn. of motion?</u>

Start with uniform random no. generator (0,1)

p(x)dx=dx for 0<x<1, =0 otherwise

y(x)-> prescribed function of x

f(y)=? (prob. of y)

f(y)dy=p(x)dx -> area under the curve for each transformed element must remain same (illustrate)

$f(y) = p(x)\frac{dx}{dy}$; $p(x)=1$: $f(y) = \frac{dx}{dy}$; $dx=f(y)dy$

$x = \int_{-\infty}^{y} f(y)dy = F(y)$

Make a table of (y,F(y)); F(y) numerically obtained

Take a uniform random number x, read from the table the corresponding y through interpolation

{x1,x2,x3,……}->{y1,y2,y3,…….}, the y-sequence wil follow f(y) (Illustrate with Gaussian)

<u>Now back to Langevin equation:</u>

$$dP/dt = -(dU/dX) - \eta(dX/dt) + R(t)$$

and $dX/dt = P/m$

$\Rightarrow$ $dP/dt = -(dU/dX) - \beta P + gG(t)$ *where* $\beta=\eta/m$ *and* $gG(t)=R(t)$

or $dP/dt = H(P(t),X(t)) + gG(t)$ *and* $\langle G(t)G(t')\rangle = 2\delta(t-t')$ *and* $g=\sqrt{(\eta T)}$

Discretizing

$$P(t+\tau) - P(t) = \int_t^{t+\tau} dt' H(t') + g \int_t^{t+\tau} dt' G(t') \cong \tau H(t)+gG_1(t)$$

$$X(t+\tau) - X(t) \cong \tau P(t)/m$$

Here $G_1(t) = \int_t^{t+\tau} dt' G(t')$ *is also a Gaussian-distributed random number*

How? $\langle G_1(t)\rangle = \int_t^{t+\tau} dt' \langle G(t')\rangle = 0$ *and*

$$\langle G^2{}_1(t)\rangle = \int_t^{t+\tau} dt_1 \int_t^{t+\tau} dt_2 \, \langle G(t_1)G(t_2)\rangle = 2\tau$$

generate Gaussian numbers ω with $\langle \omega^2 \rangle = 2$ and use $G_1(t) = \sqrt{\tau}\,\omega(t)$.

Perform integration choosing the random force at each step from sampling a Gaussian distribution.

Since random numbers are used at each time step, each Langevin trajectory will be different though started with same initial condition (X_0, P_0).

After each time step, check if $X > X_{sci}$ (scission point) or not.

If YES,

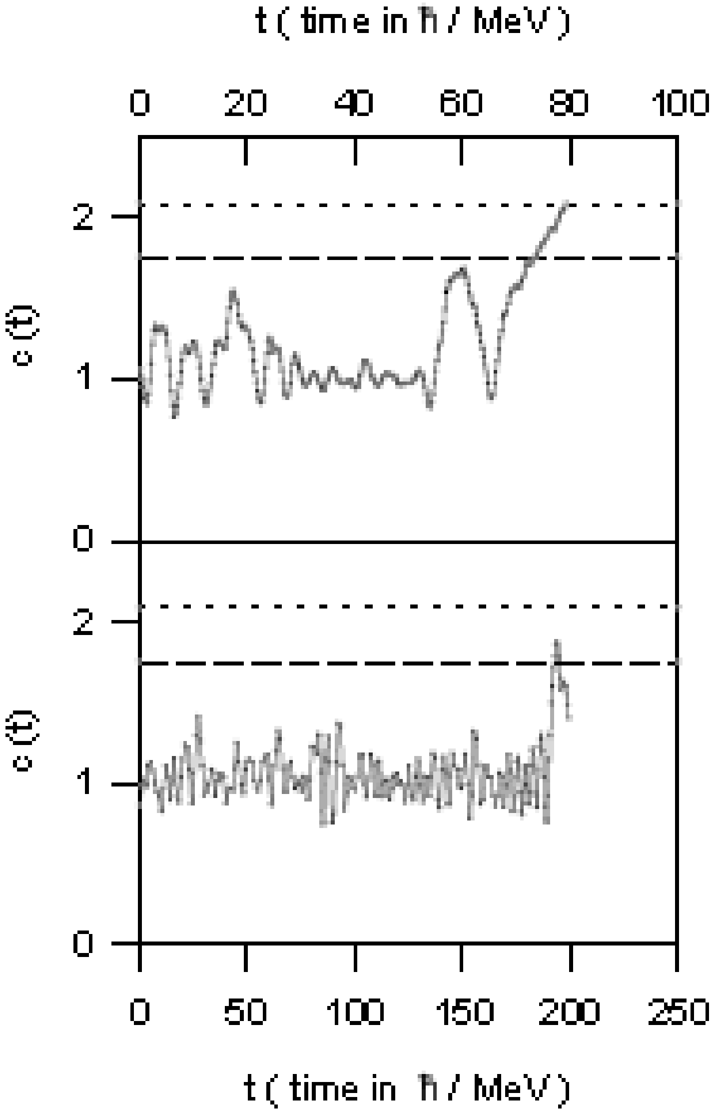
count the event as a fission event, record the instant, $n_{CN} = n_{CN} - 1$.

If NO,

continue the process (till some very large time t_{max}).

Repeat the procedure for a large number of events

Show typical plot along with potential profile



At the end of a run

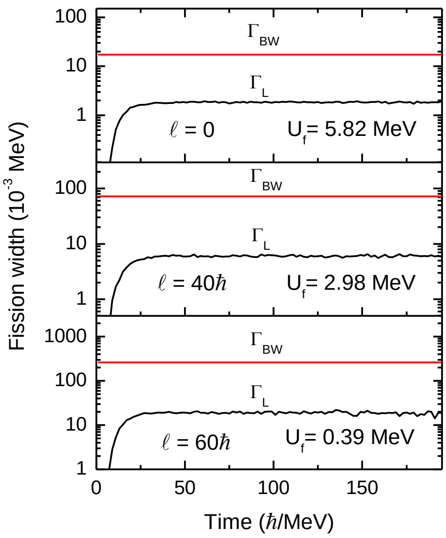
we have a distribution of life-time of fission events

$n_{CN}(t - \Delta t/2)$ = No. of CN at time $(t - \Delta t/2)$

$n_{CN}(t + \Delta t/2)$ = No. of CN at time $(t + \Delta t/2)$

Assuming law of radioactive decay, decay rate at time 't',

$$\lambda(t) = \frac{\ln\{n_{CN}(t-\Delta t/2)/n_{CN}(t+\Delta t/2)\}}{\Delta t}$$



1.Y. Abe et al., Phys. Rep. 275 (1996) 49

2. P. Frobrich & I.I.Gontchar, Phys. Rep. 292 (1998)131

Input for solving Langevin equation:

- Collective potential U ⇒ LDM
- Collective inertia m ⇒ hydro-dynamical model assuming no vortex
- Dissipation coefficient η ⇒ nuclear bulk property, one-body dissipation (sometimes treated as a parameter)

Potential (Finite Range Liquid Drop Model potential)AJ Sierk, PRC33(1986)2039

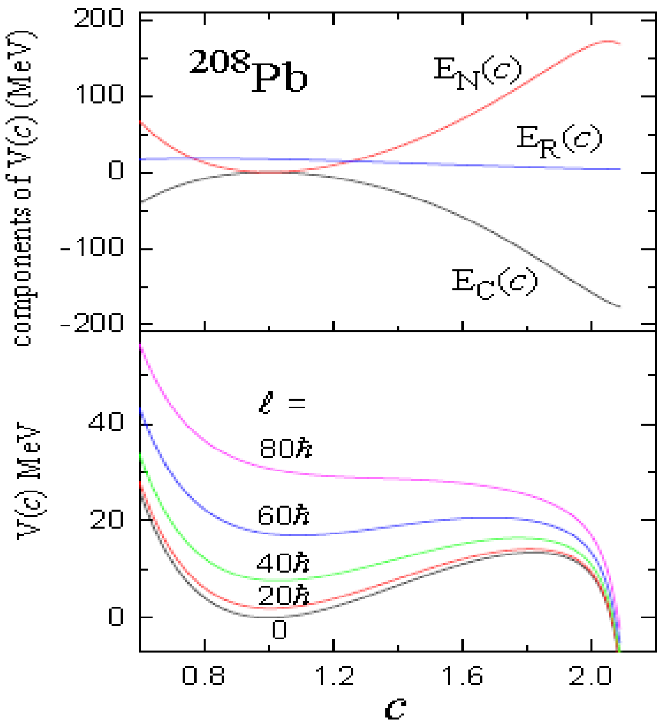
double folding of Yukawa+exponential

$$U(X(shape-parameter)=\iint d^3r_1d^3r_2\rho(r_1)\rho(r_2)v_{eff}(r_1-r_2)$$

Illustrate with figure

Parameters of v_{eff} fixed by fitting fission barriers of heavy nuclei

Add Coulomb + rotational



Discuss only the lower plot

Collective inertia

Nucleus $\rightarrow$ incompressible and irrotational (no vortex) fluid (Davies , Sierk & Nix, PRC13(1976)2385)

$$T = \frac{1}{2}\rho_m \int v^2 d^3 r$$

$$\overrightarrow{v} = \sum (\partial \overrightarrow{r} / \partial q_i) \dot{q}_i.$$

q_i collective (shape) coordinate

$$T = \frac{1}{2} \sum m_{ij}(\overrightarrow{q}) \dot{q}_i \dot{q}_j.$$

m_{ij} -> analytically obtained

One-body dissipation

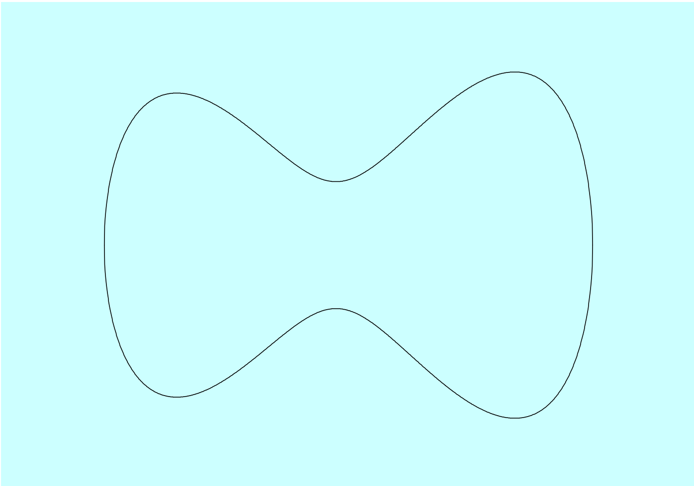
Wall formula

Particle hits moving wall (Brownian particle)

Receives kick from wall

Wall motion slows down

Dissipation in wall motion results



$$-\dot{E}^{wall} = \rho_m \overline{v} \oint \dot{n}^2 ds,$$

$$-\dot{E}_{dis} = \sum_{i,j} \eta_{ij}(\overrightarrow{q})\dot{q}_i\dot{q}_j,$$

Analytical form of η obtained

J. Blocki et al., Ann. Phys.113 (1978) 330

J.Randrup & W.J. Swiatecki, Ann. Phys.125 (1980)193

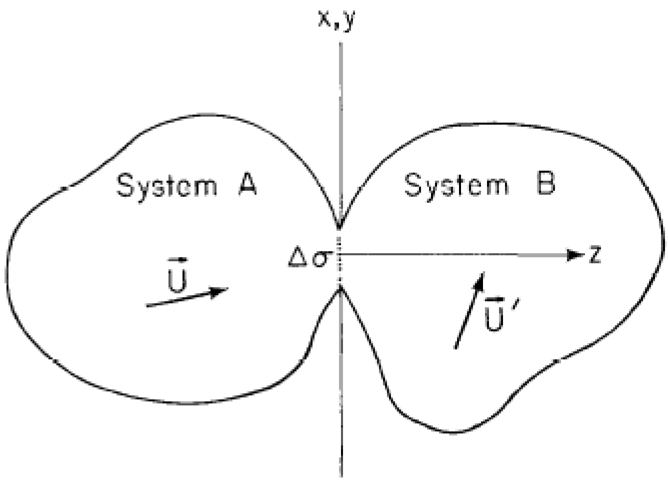
<u>Window dissipation</u>

Transfer of particles$\rightarrow$ transfer of momentum

Irreversible

Net effect $\rightarrow$ dissipation

Effective only when a window is open (neck is formed)



Classical expression $\rightarrow$

$$-\dot{E}^{window} = \frac{1}{2}\rho_m \overline{v} \triangle \sigma u^2 .$$

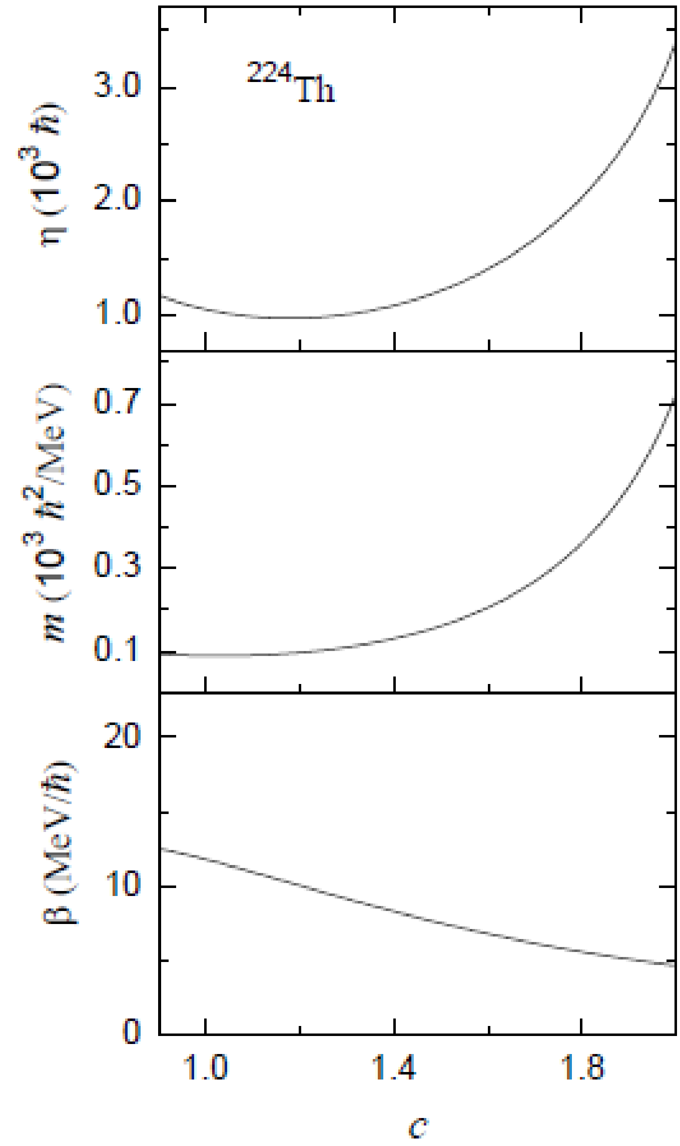
U $\rightarrow$ relative speed between left and right pieces

$$u = \dot{R} = \sum (\partial R/\partial q_i)\dot{q}_i$$

$$\eta_{ij}^{window}(\overrightarrow{q}) = \frac{1}{2}\rho_m \overline{v} \triangle \sigma \frac{\partial R}{\partial q_i}\frac{\partial R}{\partial q_j}.$$

Blocki et al. Ann.of Phys.113,330(1978)

Inertia, dissipation co-efficient depend on nuclear shape



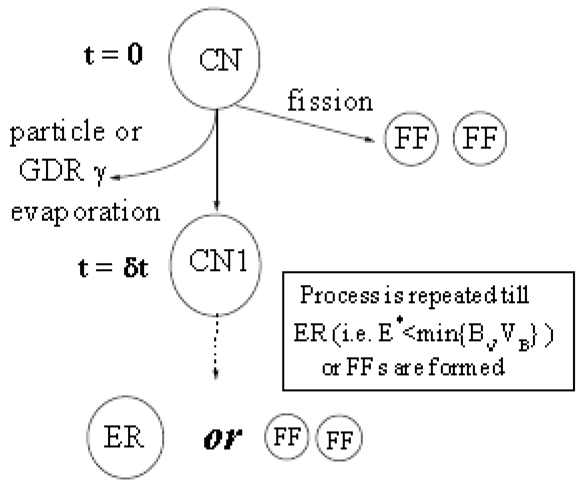
Sadhukhan & Pal:Phys.Rev.C82, 021601(R)(2010)

How to couple particle emission in a Langevin dynamical calculation?

During each time-step of Langevin integration, consider emission of particles also

<u>Algorithm for compound nuclear (CN) decay</u>

<u>Fission-> Langevin dynamics; evaporation->statistical</u>



Start with a CN(E_X, l)

Solve Langevin eqn. for time step Δt

If fission occurs, stop (count as a fission event)

If not, do a Monte-Carlo sampling to decide if the CN has decayed in Δt

If YES, decide decay type (n,p,α,γ) by another Monte-Carlo

Re-adjust (A, E_X, l) of residual nucleus and continue

Otherwise $\rightarrow$ go to the next time step and continue

<u>How to Monte-Carlo a decay probability?</u>

We shall assume the radioactive law of CN decay

i.e. Probability of a CN to decay in Δt $\propto \Delta$t

$= r\Delta t \quad = (\Gamma/\hbar)\Delta t$

Here, Γ is the total width of a CN decay given as

$\Gamma = \Gamma_n + \Gamma_p + \Gamma_\alpha + \Gamma_\gamma$ etc.

Underlying assumption: all the processes are independent

<u>Particle/gamma decay widths (Feshbach formula)</u> (Puhlhofer;NPA280,278(1977))

All the decay widths depend upon excitation energy and spin of the CN

$$\Gamma_i = \Gamma_i (E_x, I)$$

Probability of a CN to decay in Δt = p= $(\Gamma/\hbar)\Delta t < 1$ and constant

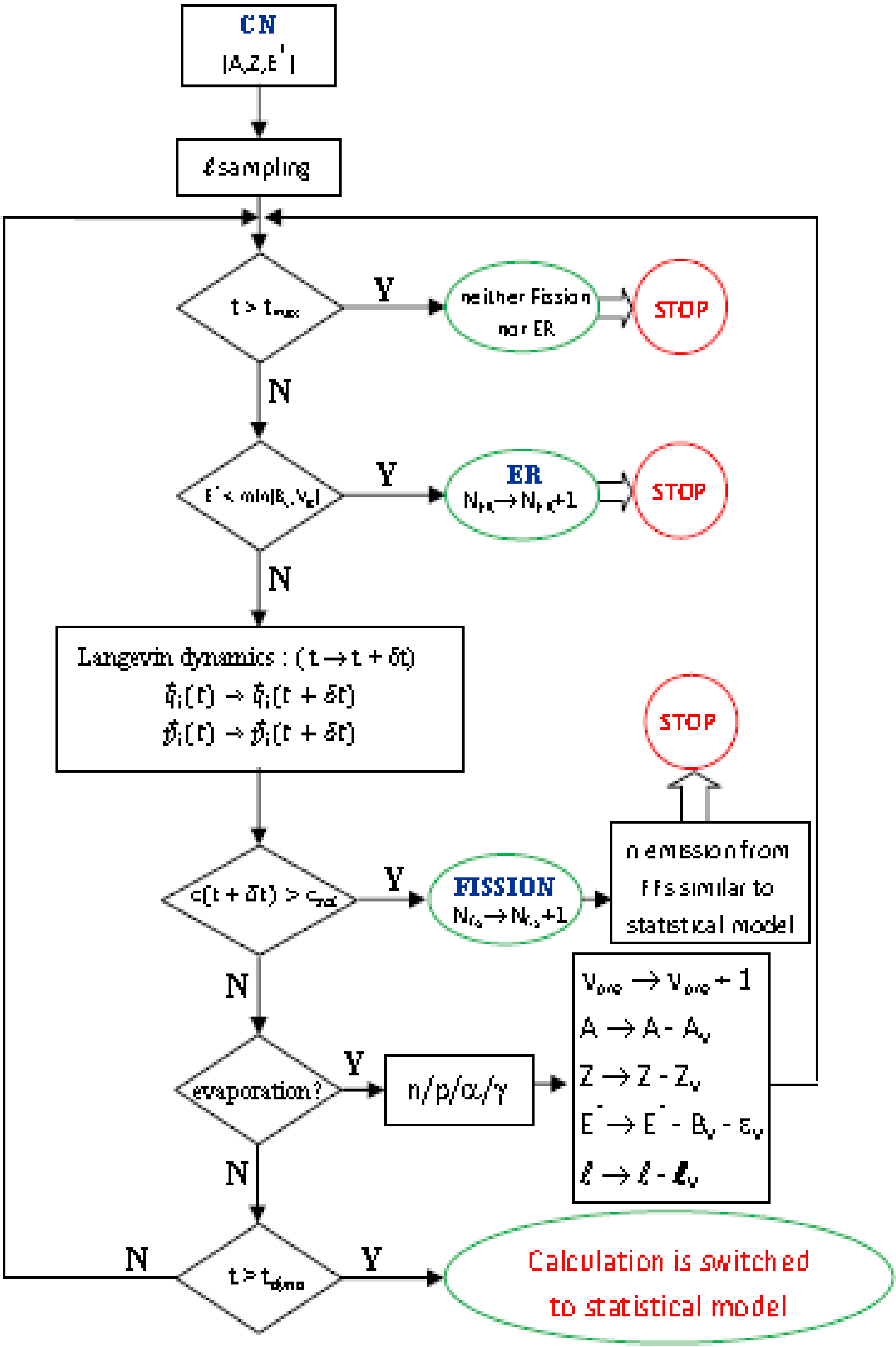
So we do a uniform sampling

We call a subroutine which generates uniformly distributed random numbers in the range 0 to 1 , output$\rightarrow$ r

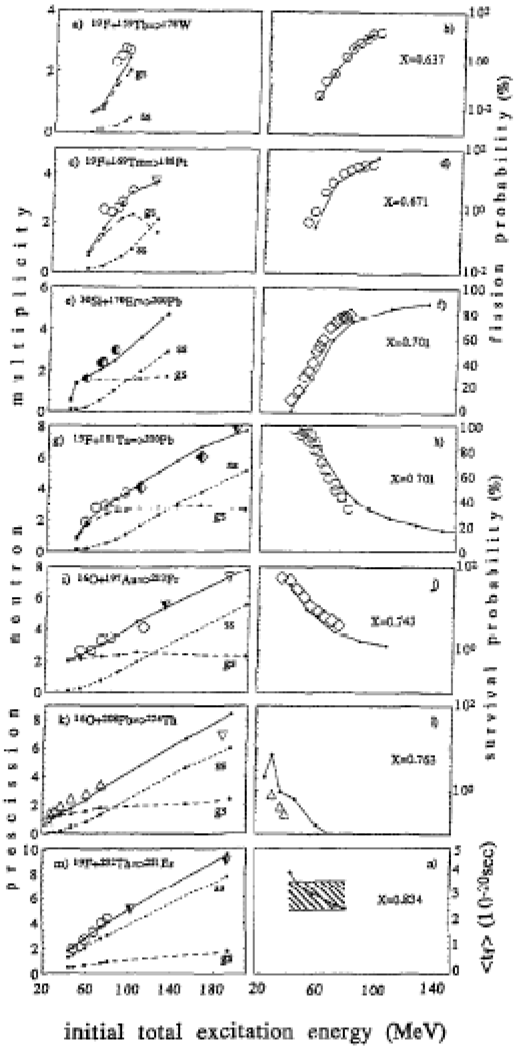
If $r \leq p \rightarrow$ CN has decayed in Δt

If $r > p \rightarrow$ CN has survived Δt

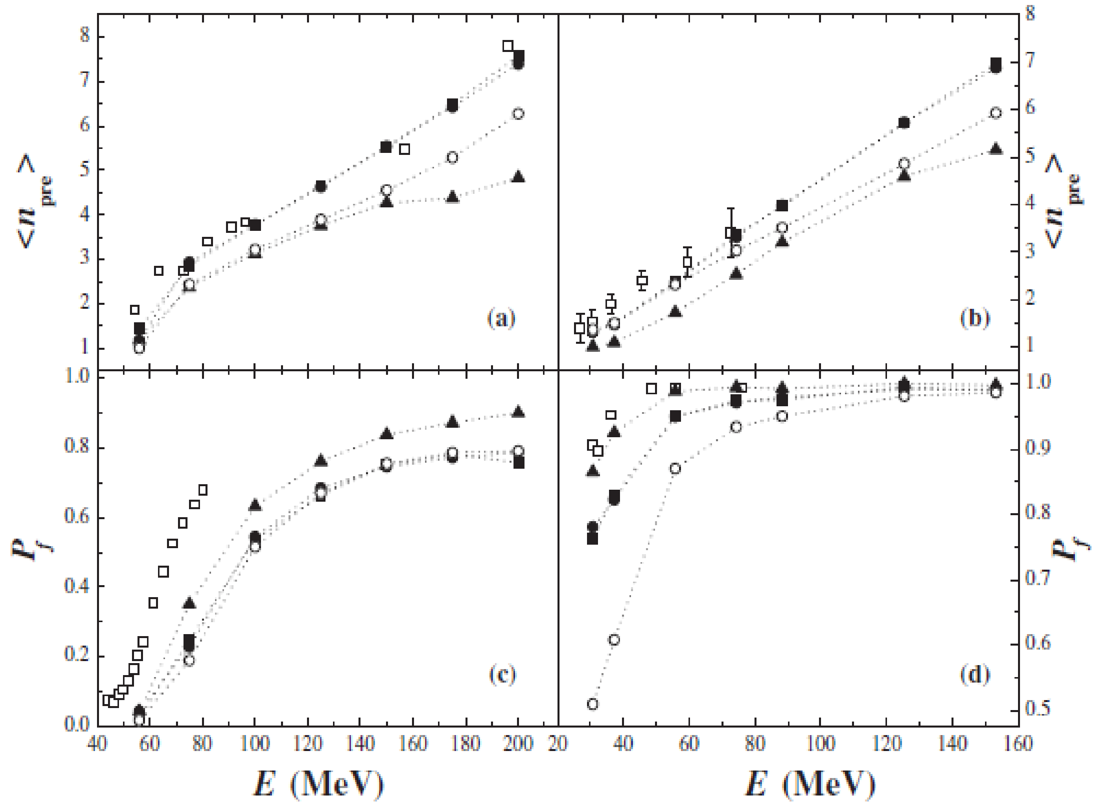
If it decays, the type of decay (i.e. f or n or p or α or γ) can also be decided by uniform sampling of partial widths (Γ_i/Γ)



CN
(A,Z,E*)
ℓ sampling
t > t_max
Y
neither Fission nor ER
STOP
N
E* < min(B_ν, V_B)
Y
ER
N_ER → N_ER+1
STOP
N
Langevin dynamics : (t → t + δt)
q_i(t) → q_i(t + δt)
p_i(t) → p_i(t + δt)
STOP
c(t + δt) > c_sc
Y
FISSION
N_fis → N_fis+1
n emission from FFs similar to statistical model
N
ν_pre → ν_pre + 1
A → A - A_ν
Z → Z - Z_ν
E* → E* - B_ν - ε_ν
ℓ → ℓ - ℓ_ν
evaporation?
Y
n/p/α/γ
N
N
t > t_dyna
Y
Calculation is switched to statistical model



Frobrich,Gontchar;NPA563(1993)326



Karpov et al.,J.Phys.G.Nucl.Phys.29,2365(2003)

□→ experimental data
Other symbols→ different 'a'

3-D results

Karpov et al. PRC63,054610(2001)

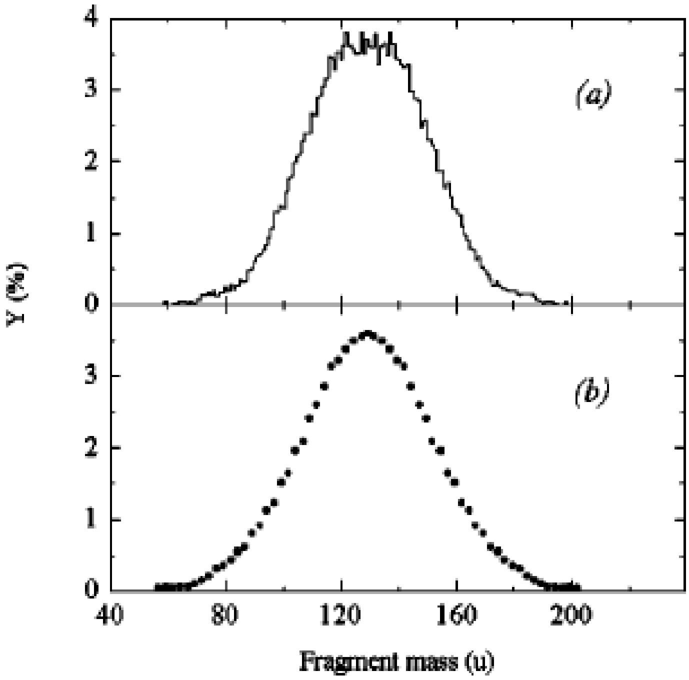


FIG. 8. The theoretical (a) and experimental (b) mass distributions of fission fragments of ^{260}Rf, $E^* = 74.2$ MeV. The theoretical histogram was calculated with the reduction coefficient $k_s = 0.1$. The experimental distribution was taken from Ref. [57].

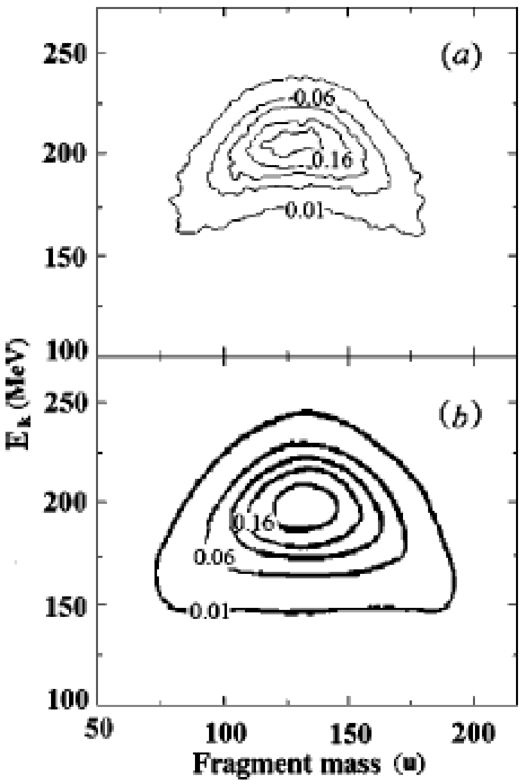
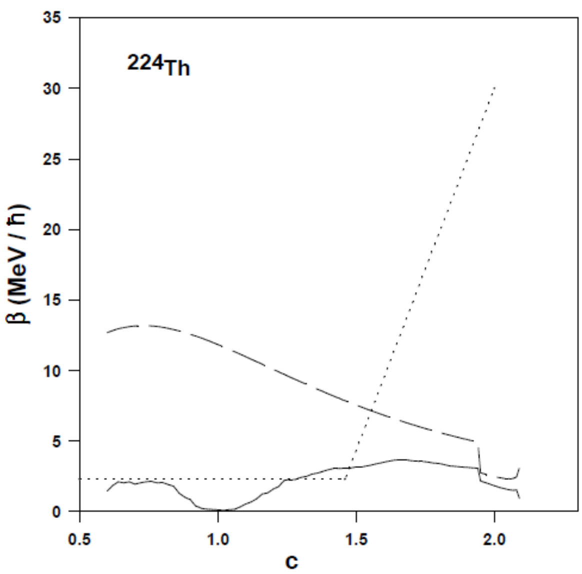
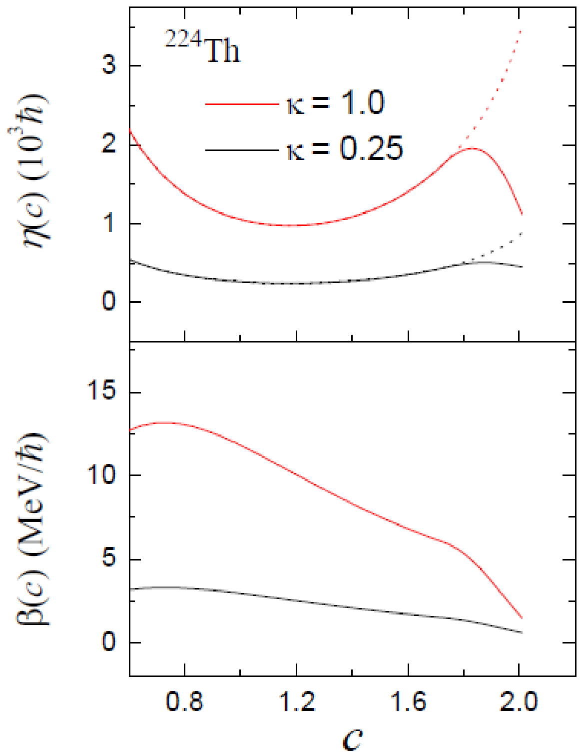


FIG. 6. The theoretical (a) and experimental (b) MED of fission fragments of ^{260}Rf at the total excitation energy $E^*=74.2$ MeV. The numbers at the contour lines in percents indicate the yield, which is normalized to 200%. The theoretical diagram was calculated with the reduction coefficient $k_s=0.1$. The experimental diagram was taken from Ref. [57].



Frobrich et al.(1993)



Karpov et al.(2001)

<u>Wall formula too strong to reproduce experimental data:</u>

A reduction factor seems necessary

Why a reduction factor?

$$\eta_{CWWF} = \mu \eta_{WF}$$

Magnitude of η is an open problem

Often used as a fit parameter

Alternative approach to stochastic dynamics $\Rightarrow$

Fokker-Planck equation

- ❑Consider the total ensemble of Langevin trajectories

The evolution of the ensemble with time can be viewed as a diffusion process

In stead of individual trajectories, we can discuss in terms of a probability distribution function $\rho(X,P,t)$

$\rho(X,P,t)dXdP \Rightarrow$ *probability of finding a CN with collective coordinate and momentum in the range*

$X \rightarrow X+dX$ and $P \rightarrow P+dP$ at time 't'.

Fokker-Planck equation from Langevin equation

Liouville's theorem:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial X}\dot{X}\rho + \frac{\partial}{\partial P}\dot{P}\rho = 0$$

density conservation

$$\frac{dP}{dt} = -\frac{dU}{dX} - \eta\frac{dX}{dt} + R(t)$$
$$\frac{dX}{dt} = \frac{P}{m}$$

Langevin eq.

$$\frac{\partial\rho}{\partial t} = -\left(\frac{\partial}{\partial X}\dot{X} + \frac{\partial}{\partial P}\dot{P}\right)\rho(X,P,t)$$
$$= \Omega(X,P,t)\rho(X,P,t)$$
$$\Omega(X,P,t) = \frac{\partial}{\partial P}\left(\frac{dU}{dX} + \frac{\eta}{m}P - R(t)\right) - \frac{\partial}{\partial X}\left(\frac{P}{m}\right)$$

$$\rho(X,P,t+\Delta t) = \rho(X,P,t) + \int_t^{t+\Delta t}\Omega(X,P,t_1)\rho(X,P,t_1)dt_1$$

$$= \rho(X,P,t) + \int_t^{t+\Delta t} dt_1\Omega(X,P,t_1)\left[\rho(X,P,t) + \int_t^{t_1}\Omega(X,P,t_2)\rho(X,P,t_2)dt_2\right]$$
$$= \left[1 + \int_t^{t+\Delta t} dt_1\Omega(X,P,t_1) + \int_t^{t+\Delta t} dt_1\Omega(X,P,t_1)\int_t^{t_1} dt_2\Omega(X,P,t_2) + ...\right]\rho(X,P,t)$$

Δt >> time-scale of R(t)

$$\langle R(t)\rangle = 0$$
$$\langle R(t)R(t')\rangle = I_R\delta(t-t')$$
$$I_R = 2\eta T$$

$$\int_t^{t+\Delta t} dt_1\Omega(X,P,t_1) = \Delta t\langle\Omega(X,P,t)\rangle = \Delta t\left\langle\frac{\partial}{\partial P}\left(\frac{dU}{dX} + \frac{\eta}{m}P - R(t)\right) - \frac{\partial}{\partial X}\left(\frac{P}{m}\right)\right\rangle$$
$$= \Delta t\left[\ \frac{\partial}{\partial P}\left(\frac{dU}{dX} + \frac{\eta}{m}P\right) - \frac{\partial}{\partial X}\left(\frac{P}{m}\right)\ \right]$$

Remembering time-average->ensemble average

$$\int_{t}^{t+\Lambda t}\int_{t}^{t_1} dt_1 dt_2 \Omega(t_1)\Omega(t_2) = \frac{1}{2}\int_{t}^{t+\Delta t}\int_{t}^{t+\Delta t} dt_1 dt_2 \Omega(t_1)\Omega(t_2)$$

$$\int_{t}^{t+\Delta t}\int_{t}^{t+\Delta t}\Omega(t_1)\Omega(t_2)dt_1 dt_2 = \iint \begin{matrix} \left\{\frac{\partial}{\partial P}\left(\frac{dU}{dX}+\frac{\eta}{m}P\right)-\frac{\partial}{\partial X}\left(\frac{P}{m}\right)-R(t_1)\frac{\partial}{\partial P}\right\}\times \\ \left\{\frac{\partial}{\partial P}\left(\frac{dU}{dX}+\frac{\eta}{m}P\right)-\frac{\partial}{\partial X}\left(\frac{P}{m}\right)-R(t_2)\frac{\partial}{\partial P}\right\}dt_1 dt_2 \end{matrix}$$

Let

$$F = \frac{\partial}{\partial P}\left(\frac{dU}{dX}+\frac{\eta}{m}P\right)-\frac{\partial}{\partial X}\left(\frac{P}{m}\right)$$

$$\mathrm{int}\, egral = \iint \left\{F - R(t_1)\frac{\partial}{\partial P}\right\}\left\{F - R(t_2)\frac{\partial}{\partial P}\right\}dt_1 dt_2$$

$$= F^2(\Delta t)^2 + \frac{\partial^2}{\partial P^2}\iint R(t_1)R(t_2)dt_1 dt_2$$

$$= F^2(\Delta t)^2 + I_R \Delta t \frac{\partial^2}{\partial P^2}$$

Substituting,

$$\rho(t+\Delta t)=\left[1+F\Delta t+\frac{1}{2}F^2(\Delta t)^2+\frac{1}{2}I_R\Delta t\frac{\partial^2}{\partial P^2}+....\right]\rho(t)$$

$$\frac{\rho(t+\Delta t)-\rho(t)}{\Delta t}=F\rho(t)+\frac{1}{2}I_R\frac{\partial^2}{\partial P^2}\rho(t)+\frac{1}{2}\Delta tF^2\rho(t)+.....$$

$$\Delta t\to 0$$

$$\frac{\partial}{\partial t}\rho(X,P,t)=\left\{-\frac{\partial}{\partial X}\frac{P}{m}+\frac{\partial}{\partial P}\frac{dU}{dX}+\frac{\partial}{\partial P}\left(\beta P+m\beta T\frac{\partial}{\partial P}\right)\right\}\rho(X,P,t)$$

$$\beta=\frac{\eta}{m}$$

Fokker-Planck eqn.,

Kramers,Physica(Amsterdam)7,284(1940)

Generalized Liouville's eqn. to include dissipation

Diffusion eqn. in phase space

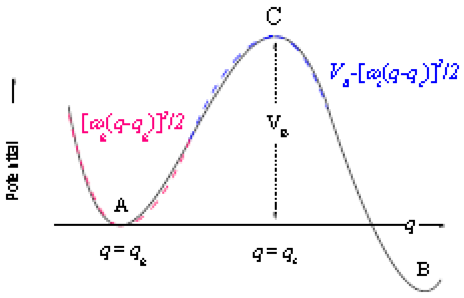
<u>Kramers' analytical solution of Fokker-Planck equation:</u>

Fokker-Planck equation in one-dimension

$$\frac{\partial\rho}{\partial t}+\frac{P}{m}\frac{\partial\rho}{\partial X}-\frac{dU}{dX}\frac{\partial\rho}{\partial P}=\beta\frac{\partial(P\rho)}{\partial P}+m\beta T\frac{\partial^2\rho}{\partial P^2}$$

$$U=\frac{1}{2}m\omega_g^2(X-X_g)^2$$

$$=V_B-\frac{1}{2}m\omega_s^2(X-X_s)^2$$



- Ensemble of large number of CN at A
- Weak diffusion current at C (V_B > T)$\rightarrow$ density at A does not change
- Steady state$\rightarrow\partial\rho/\partial t$ = 0

Steady state F-P eqn.

$$\frac{P}{m}\frac{\partial \rho}{\partial X} - \frac{dU}{dX}\frac{\partial \rho}{\partial P} = \beta\frac{\partial (P\rho)}{\partial P} + m\beta T\frac{\partial^2 \rho}{\partial P^2}$$

Desired solution$\rightarrow$

At A $\rightarrow$ Boltzmann

$$\rho = K\exp\left[-\frac{\frac{P^2}{2m}+U}{T}\right]$$

$$\rho = K \exp\left[-\frac{\frac{P^2}{2m} + \frac{1}{2} m\omega_g^2 (X - X_g)^2}{T} \right]$$ satisfies F-P (Spl.soln)

At C$\rightarrow$ With modification

At B$\rightarrow$ Zero

General solution

$$\rho = KF(X,P)\exp\left[-\frac{\frac{P^2}{2m} + U}{T} \right]$$

$$\rho = KF(X,P)e^{-V_B/T} \exp\left[-\frac{\frac{P^2}{2m} - \frac{1}{2} m\omega_s^2 (X - X_s)^2}{T} \right]$$

F(X,P) $\approx$1 at X=X_g

$\approx$0 at X>>X_s

Aim is to find F

Re-define X=X-X_s

$$m\beta T \frac{\partial^2 F}{\partial P^2} = \frac{P}{m}\frac{\partial F}{\partial X} + \frac{\partial F}{\partial P}\left(m\omega_s^2 X + \beta P\right)$$

Solution exists if we assume ς=P-aX (Zeta)

F(X,P)=F(ς)

$$m\beta T\frac{d^2F}{d\varsigma^2}=-\left(\frac{a}{m}-\beta\right)\left(P-\frac{m\omega_s^2}{\frac{a}{m}-\beta}X\right)\frac{dF}{d\varsigma}$$

We must have $$a=\frac{m\omega_s^2}{\frac{a}{m}-\beta}$$

$$m\beta T\frac{d^2F}{d\varsigma^2}=-\left(\frac{a}{m}-\beta\right)\varsigma\frac{dF}{d\varsigma}$$

Solution:

$$F(\varsigma)=\sqrt{\frac{1}{2\pi m\beta T}\left(\frac{a}{m}-\beta\right)}\int_{-\infty}^{\varsigma}e^{-\frac{1}{2m\beta T}\left(\frac{a}{m}-\beta\right)\varsigma^2}d\varsigma$$

F(ζ)=0 for $\zeta\to-\infty$ implying a=+ve when X$\to\infty$ at far right of saddle

F(ζ)=1 for $\zeta\to+\infty$ implying a=+ve when X$\to-\infty$ at far left of saddle (May not hold at large spin)

'a' should be positive

Defining eqn.$\rightarrow$ $$a = \frac{m\omega_s^2}{\frac{a}{m} - \beta}$$

Taking +root

$$a = \frac{m\beta}{2} + \sqrt{\omega_s^2 m^2 + \frac{m^2\beta^2}{4}}$$

Full solution: $$\rho = KF(X,P)e^{-V_B/T}\exp\left[-\frac{\frac{P^2}{2m} - \frac{1}{2}m\omega_s^2(X - X_s)^2}{T}\right]$$

With $$F(\varsigma) = \sqrt{\frac{1}{2\pi m\beta T}\left(\frac{a}{m} - \beta\right)}\int_{-\infty}^{\varsigma} e^{-\frac{1}{2m\beta T}\left(\frac{a}{m} - \beta\right)\varsigma^2}\,d\varsigma$$

Our task is to calculate the fission rate

Current across C

$$j = \int_{-\infty}^{+\infty} \rho(X = X_s, P)\frac{P}{m}dP$$

$$= KTe^{-\frac{V_B}{T}}\sqrt{\frac{a/m - \beta}{a/m}}$$

$$= KTe^{-\frac{V_B}{T}}\left\{\sqrt{1+\left(\frac{\beta}{2\omega_s}\right)^2} - \frac{\beta}{2\omega_s}\right\}$$

integrating by parts

No. of particles in pocket at ‘A’=

$$n_g = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(near - A)dPdX = \frac{2\pi KT}{\omega_g}$$

$$\Gamma_K = \hbar\frac{j}{n_g} = \frac{\hbar\omega_g}{2\pi}e^{-\frac{V_B}{T}}\left\{\sqrt{1+\left(\frac{\beta}{2\omega_s}\right)^2} - \frac{\beta}{2\omega_s}\right\}$$